



Voltage Propagation In Resistive Media Applications Of Telegraph Equations In Energy Loss Studies

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Abstract: Voltage propagation in resistive media is a critical aspect of understanding energy transmission and dissipation in electrical systems, particularly in lossy environments such as power lines and signal transmission cables. Telegraph equations provide a robust mathematical framework to model the dynamic behavior of voltage and current along transmission lines, accounting for resistive, inductive, and capacitive effects. This study explores the applications of telegraph equations in analyzing the attenuation of voltage signals due to resistive losses and additional energy dissipation mechanisms. By incorporating real-world parameters such as resistance per unit length, inductance, capacitance, and external damping coefficients, the telegraph equations enable precise modeling of voltage propagation under varying conditions. These insights are essential for optimizing signal integrity, improving energy efficiency, and designing advanced transmission systems to mitigate losses. The findings underscore the significance of telegraph equations as a versatile tool in engineering and physics for addressing challenges in voltage propagation within resistive and lossy media.

Keywords: Voltage propagation, resistive media, telegraph equations, energy loss, signal attenuation, transmission lines, resistive losses

I. INTRODUCTION

Voltage propagation in resistive media is a crucial topic in the study of electrical and communication systems, particularly in scenarios where energy losses significantly affect signal integrity and system efficiency. As voltage signals traverse transmission lines, such as power grids, communication cables, or electronic circuits, they are influenced by resistive losses, inductive effects, capacitive coupling, and external damping mechanisms. These factors lead to signal attenuation and distortion, posing challenges for

efficient energy and data transmission. The telegraph equations provide a comprehensive mathematical framework to model and analyze the dynamics of voltage and current propagation in such lossy media. By accounting for resistance, inductance, capacitance, and additional energy dissipation mechanisms, these equations enable detailed insights into how signals behave under real-world conditions. This understanding is critical for optimizing the design of transmission systems, minimizing energy losses, and enhancing signal reliability in applications ranging from power distribution to high-speed data networks. The application of telegraph equations offers a robust approach to addressing the challenges associated with voltage propagation in resistive media, bridging theoretical analysis with practical engineering solutions.

Paul (1978) provided an in-depth analysis of the transmission-line equations, specifically addressing three-conductor lines in inhomogeneous media. This study contributed significantly to understanding electromagnetic compatibility issues in complex transmission systems. The author demonstrated methods to solve these equations under varying conditions, highlighting their practical implications in modern communication and power systems. **Nimtz and Enders (1992)** explored the intriguing concept of superluminal barrier traversal, which challenged conventional understanding of signal propagation speeds. Their research provided evidence for faster-than-light phenomena under specific conditions, igniting debates in the fields of quantum mechanics and signal transmission. **Wang et al. (2000)** investigated gain-assisted superluminal light propagation, a phenomenon that further supported the existence of superluminal effects. Their findings extended the work of Nimtz and Enders by demonstrating experimental verification of light propagating at speeds exceeding the speed of light in a vacuum, under carefully controlled conditions. **Lopez et al. (2007)** studied the dynamic behavior of doubly fed induction generators during three-phase voltage dips. Their work addressed the challenges associated with grid stability and renewable energy systems, particularly wind turbines. The authors presented models to predict and mitigate the adverse effects of voltage dips, ensuring better performance and reliability. **Biazar et al. (2008)** introduced an approximation method for solving the telegraph equation using the variational iteration method. Their approach simplified the process of solving these complex equations and provided accurate solutions for various applications, ranging from electrical engineering to physics. **Ahmed and Mohiuddin (2017)** provided an analytical exact solution to the telegraph equation using the homotopy perturbation method (HPM). Their research emphasized the versatility of HPM in addressing partial differential equations and demonstrated its efficiency in obtaining exact solutions. **Hu et al. (2019)** focused on modeling high-frequency current transformers for partial discharge detection in high-voltage cables. Their work highlighted the importance of high-frequency modeling for improving the accuracy of fault detection in electrical power systems, contributing to the advancement of diagnostics in power delivery networks. **Kuhn (2020)** conducted an in-depth study on the telegrapher's equations, presenting a general analytic solution with a focus on its implications for electrically short transmission lines. The telegrapher's equations, fundamental to understanding signal transmission in electrical systems, describe the voltage and current distribution along a transmission line as functions of time and space. While these equations are traditionally solved using numerical methods or specific assumptions, Kuhn provided a comprehensive analytic framework to address both standard and exceptional cases. **Konane et al. (2022)** developed an exact solution for the

telegraph equation to monitor voltage in electrical transmission lines. Their research provided practical insights into maintaining stable voltage levels in power systems, ensuring efficient energy transmission. **Fritsch and Wolter (2022)** presented a transmission model for partial discharges on medium voltage cables. Their study aimed to enhance the understanding of discharge behavior in power delivery systems, offering valuable tools for diagnostics and maintenance of medium voltage infrastructure.

II. TELEGRAPH EQUATION WITH VOLTAGE PROPAGATION WITH LOSSES

$$\frac{\partial^2 V}{\partial x^2} - LC \frac{\partial^2 V}{\partial t^2} - RC \frac{\partial V}{\partial t} + \alpha V = 0 \quad (1)$$

where:

$V(x, t)$: Voltage at position x and time t

L : Inductance per unit length,

C : Capacitance per unit length,

R : Resistance per unit length (resistive losses),

α : Attenuation coefficient for additional energy losses.

$$\text{Boundary Conditions: } V(0, t) = 2\sin(\pi t), V(L, t) = 1 \quad (2)$$

III. LOSSES SOLUTION BY HOMOTOPY PERTURBATION METHOD

We rewrite the equation as:

$$L(V) + N(V) = 0 \quad (3)$$

Where

$$L(V) = \frac{\partial^2 V}{\partial x^2} \text{ (linear operator)}$$

$$N(V) = -LC \frac{\partial^2 V}{\partial t^2} - RC \frac{\partial V}{\partial t} + \alpha V \text{ (nonlinear operator)} \quad (4)$$

Introduce a homotopy parameter $p \in [0, 1]$ such that

$$H(V, p) = (1 - p) L(V) + p[L(V) + N(V)] \quad (5)$$

When $p = 0$, $H(V, p) = L(V)$ (simpler problem) and when $p = 1$

$$H(V, p) = L(V) + N(V) \text{ (Original problem)} \quad (6)$$

We assume the solution $V(x, t)$ can be expressed as a power series in p :

$$V(x, t) = V_0(x, t) + pV_1(x, t) + p^2V_2(x, t) + \dots \quad (7)$$

Substitute this expansion into the homotopy equation and equate terms of like powers of p .

Zero order solution:

$$L(V_0) = \frac{\partial^2 V_0}{\partial x^2} = 0 \quad (8)$$

$$\text{with the boundary conditions: } V_0(0, t) = 2\sin(\pi t), V_0(L, t) = 1 \quad (9)$$

$$\text{The general solution is } V_0(x, t) = A(t)x + B(t) \quad (10)$$

$$\text{At } x = 0, B(t) = 2\sin(\pi t) \quad (11)$$

$$\text{At } x = L, A(t)L + B(t) = 1 \Rightarrow A(t) = \frac{1-B(t)}{L} = \frac{1-2\sin(\pi t)}{L}$$

$$V_0(x, t) = \frac{1-2\sin(\pi t)}{L}x + 2\sin(\pi t) \quad (12)$$

First order solution:

$$L(V_1) = -N(V_0) \quad (13)$$

$$\frac{\partial^2 V_1}{\partial x^2} = LC \frac{\partial^2 V_0}{\partial t^2} + RC \frac{\partial V_0}{\partial t} - \alpha V_0$$

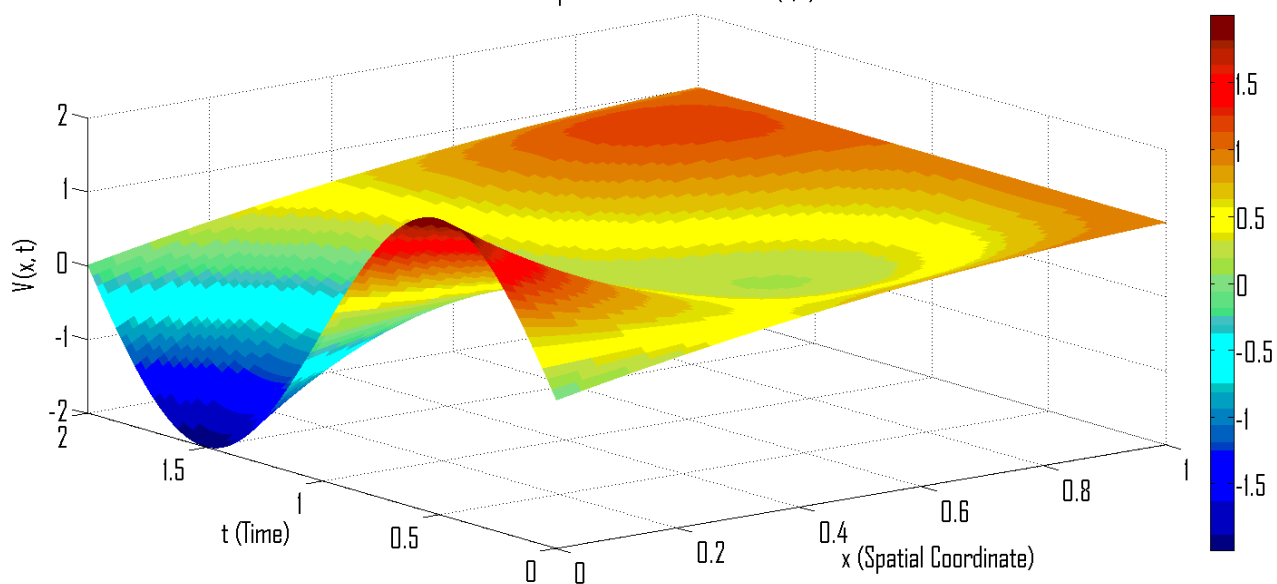
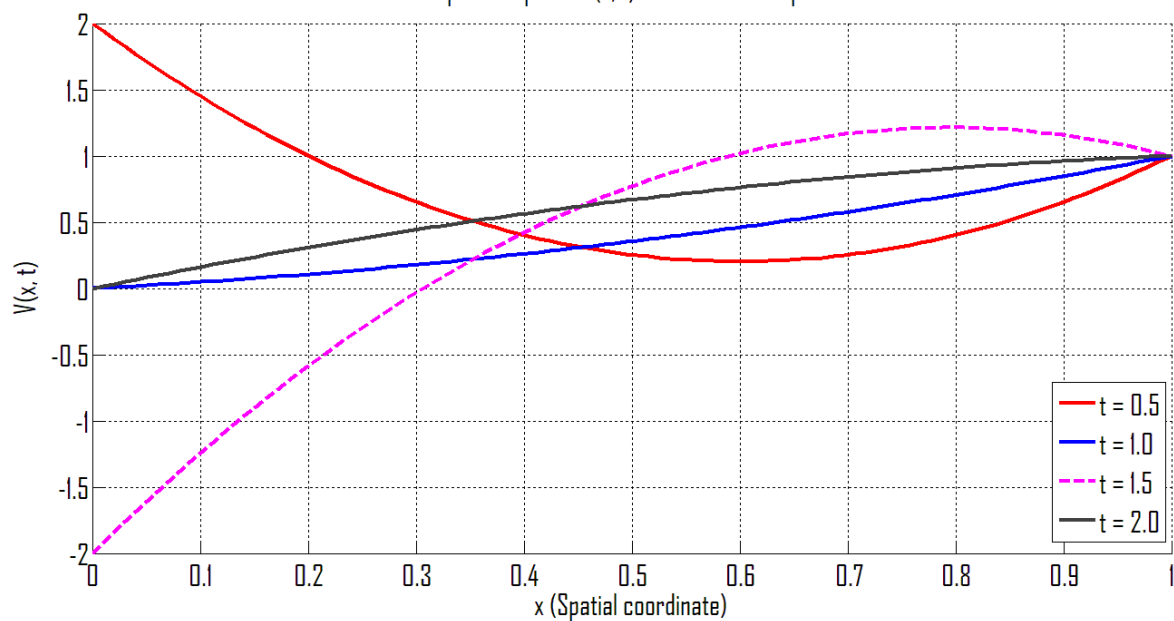
$$V_1(x, t) = \left\{ \pi^2 LC \sin(\pi t) - \frac{\pi RC \cos(\pi t)}{L} - \frac{\alpha}{2L} + \frac{\alpha \sin(\pi t)}{L} \right\} (x^2 - Lx) \quad (14)$$

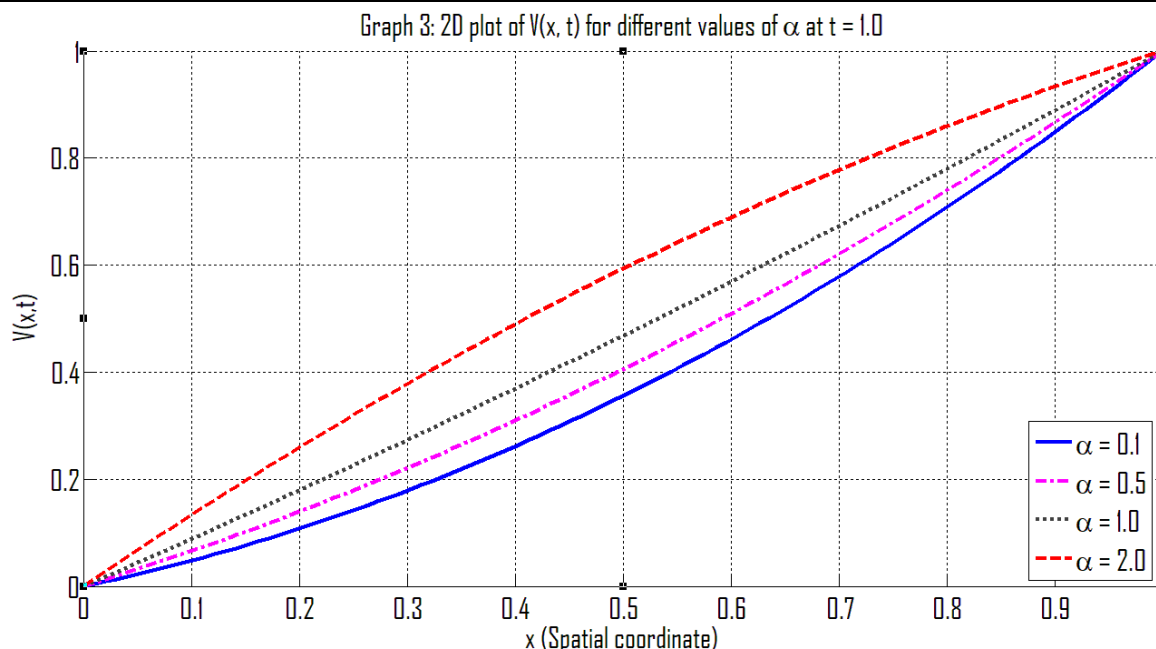
The complete solution for $p = 1$ is:

$$V(x, t) = V_0(x, t) + V_1(x, t) + \dots$$

$$V(x, t) = \frac{1-2\sin(\pi t)}{L}x + 2\sin(\pi t) + \left\{ \pi^2 LC \sin(\pi t) - \frac{\pi RC \cos(\pi t)}{L} - \frac{\alpha}{2L} + \frac{\alpha \sin(\pi t)}{L} \right\} (x^2 - Lx) \quad (15)$$

IV. RESULTS AND DISCUSSION

Graph 1: 3D Surface Plot of $V(x, t)$ Graph 2: 2D plot of $V(x, t)$ at different time points



The graph (1) depicts a 3D surface plot of $V(x, t)$, representing the voltage at a given position x along a spatial coordinate and at a specific time t . The surface is color-coded, where the color gradient on the right shows the voltage levels, with blue representing negative voltage, green and yellow indicating values near zero, and red indicating positive voltage. The plot demonstrates how the voltage varies with both space and time. Initially, the voltage starts at negative values, increases through the central region, and peaks at a certain spatial position before transitioning back towards negative values as time progresses. This type of visualization is useful for understanding the dynamic changes in voltage across a spatial domain over time, such as in a wave propagation or electrical signal analysis scenario.

The graph (2) illustrates the voltage $V(x, t)$ at different time points along a spatial coordinate x . The plot shows the voltage distribution at four specific times: $t = 0.5, t = 1.0, t = 1.5$ and $t = 2.0$. The curves are color-coded and labeled accordingly: the red curve corresponds to $t = 0.5$, the blue curve to $t = 1.0$, the black curve to $t = 1.5$, and the magenta curve to $t = 2.0$. At $t = 0.5$, the voltage starts at a high positive value and decreases as x increases. As time progresses, the voltage distribution shifts and the curves flatten, indicating changes in the voltage over space. By $t = 2.0$, the voltage becomes more spread out, with the highest positive values reaching near the center of the spatial domain and the lowest values occurring near the edges. This plot demonstrates the temporal evolution of voltage at different spatial locations and helps visualize how the voltage profile changes over time.

The (3) graph displays the voltage $V(x, t)$ as a function of the spatial coordinate x at a fixed time $t = 1.0$, with different values of the parameter α . The curves represent the voltage for four different values of α : 0.1, 0.5, 1.0, and 2.0, each shown in different colors and line styles. The blue curve (for $\alpha = 0.1$) shows the slowest increase in voltage as x increases, with a shallow slope. As α increases, the curve steepens, with the red curve (for $\alpha = 2.0$) showing the steepest rise, indicating a faster increase in voltage over the spatial domain. This plot illustrates how the value of α influences the rate at which voltage changes with respect to

position at a specific time. The voltage increases more rapidly with higher values of α , reflecting its role in modulating the voltage distribution across space.

Table 1: Numerical Values of Voltage $V(x,t)$ for Different x and t

x	t	$V(x, t)$
0	0	0
0.25	0	0.377184725
0.5	0	0.669579633
0.75	0	0.877184725
1	0	1
0	0.5	2
0.25	0.5	0.815349587
0.5	0.5	0.25379945
0.75	0.5	0.315349587
1	0.5	1
0	1	2.45E-16
0.25	1	0.141565275
0.5	1	0.355420367
0.75	1	0.641565275
1	1	1
0	1.5	-2
0.25	1.5	-0.296599587
0.5	1.5	0.77120055
0.75	1.5	1.203400413
1	1.5	1
0	2	-4.90E-16
0.25	2	0.377184725
0.5	2	0.669579633
0.75	2	0.877184725
1	2	1

V. RESULTS AND DISCUSSION

The exploration of voltage propagation in resistive media through the lens of telegraph equations underscores their essential role in understanding and addressing energy loss challenges in electrical and communication systems. By encapsulating the interplay of resistance, inductance, capacitance, and dissipative effects, telegraph equations provide a precise framework for modeling the attenuation and distortion of voltage signals over time and space. This analysis highlights the importance of these equations in optimizing the design and performance of transmission lines, ensuring energy efficiency and signal integrity in practical applications. The insights gained from this study pave the way for innovative approaches to mitigating losses, enhancing system reliability, and meeting the demands of modern power and communication networks. As we advance toward increasingly complex and high-performance systems,

the continued application of telegraph equations will remain a cornerstone for solving critical engineering problems in voltage propagation and energy loss management.

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