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Fourier Series Involving the I function of 'r' variables

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Abstract: - In this study, we evaluate three Fourier series using an integral using sine function, exponential function, the product of the Kampe De Feriet functions and the I-function of 'r' variable specified by Prathima (2). Which is of interest itself and many find significance usage in the field of applied Mathematics. Several researchers finding can be combined on specializing parameters. We also access a number of integrals. Utilizing the I- function of 'r' variable in order to obtain a multiple exponential Fourier series a Mathematical series.

Keywords: Fourier series, I-function of 'r' variables, Generalized hypergeometric functions, Mellin barnes integral.

1. Introduction: -

Mishra (1990) analyzed the following integral in this paper.

$$\int_0^{\pi}(\sin x)^{w-1}\, e^{imx}\, {}_pF_q\left[\begin{matrix}\alpha_p;\\ \beta_q;\end{matrix} C\,(\sin x)^{2h}\right]dx = \frac{\pi\, e^{im\pi/2}}{2^{w-1}}\sum_{r=0}^{\infty}\frac{(\alpha_\rho)_r\, C'\,\Gamma(w+2hr)}{(\beta_q)_r r!4^{hr}\,\Gamma\left(\frac{w+2hr\pm M+1}{2}\right)} \quad (1.1)$$

Where $(\alpha)_p$ denotes $\alpha_1,\ldots,\alpha_p;\Gamma(a\pm b)$ represents $\Gamma(a+b).\Gamma(a-b);h$ is a positive integer $;p>q$ and $Re(w)>0.$ Recall the following elementary integrals:

$$\int_0^{\pi} e^{(m-n)\pi}dx = \begin{cases}\pi, m=n;\\ 0, m\neq n;\end{cases} \quad (1.2)$$

$$\int_0^{\pi} e^{imx}\cos nx\;dx = \begin{cases}\pi/2, & m=n\neq 0;\\ \pi, & m=n=0;\\ & 0, m\neq 0;\end{cases} \quad (1.3)$$

$$\int_0^{\pi} e^{imx}\sin nx\;dx = \begin{cases}i\pi/2, & m=n.\\ & 0, m\neq 0;\end{cases} \quad (1.4)$$

Provided either both m and n are odd, or both m and n are even integers. Mishra Employed (1.2), (1.3) and (1.4) to establish three Fourier series for the product of generalized hyper geometric functions. She has also evaluated a double integral and a double Fourier exponential series for Fox's H-function.

We evaluate an integral using the product of the Kampe De Feriet functions and the I function of 'r' variable in this document and apply it is to generate three Fourier series involving the I- function of 'r' variable.

A multiple integral involving I function of ‘r’ variable is also evaluated. Derive a multiple Fourier series utilizing the above I- function of ‘r’ variable.

Finally, we also discuss some particular cases.

For brevity, we shall use the following notations:

$$\frac{\prod_{=1}^{E}(e_k)_{r+t}\prod_{k=1}^{F}(f_k)_r\prod_{k=1}^{F'}(f_k')_t}{\prod_{k=1}^{G}(gk)_{r+t}\prod_{k_1=1}^{F_1}(f_{1_{k_1}})_{r_1}\prod_{k=1}^{H'}(h_k')_t}=\in$$

$$\frac{\prod_{k_1=1}^{E_1}(e_{1_{k_1}})_{r_1+t_1}\prod_{k_1=1}^{F_1}(f_{1_{k_1}})_{r_1}\prod_{k_1=1}^{F'}\left(f_{1_{k_1}}'\right)_{t_1}}{\prod_{k_1=1}^{G}(g_1k_1)_{r_1+t_1}\prod_{k_1=1}^{H_1}(h_{1_{k_1}})_{r_1}\prod_{k_1=1}^{H'}\left(h_{k_1}'\right)_{t_1}}=\in_1$$

…. …. …. ….

$$\frac{\prod_{k_n=1}^{E_n}(e_{n_{k_n}})_{r_n+t_n}\prod_{k_n=1}^{F_n}(f_{n_{k_n}})_{r_n}\prod_{k_n=1}^{F_n'}\left(f_{n_{k_n}}'\right)_{t_n}}{\prod_{k_n=1}^{G_n}(g_nk_n)_{r_n+t_n}\prod_{k_n=1}^{H_n}(h_{n_{k_n}})_{r_n}\prod_{k_n=1}^{H_n'}\left(h_{n_{k_n}}'\right)_{t_n}}=\in_n.$$

2. Main Integral:

In this section, making as appeal to (1.1), we derive

$$\int_0^{\pi}(\sin x)^{w'-1}e^{imx}F_{G:H;H'}^{E:F;F'}\left[\begin{matrix}(e):(f);(f')\\(g):(h);(h')\end{matrix};\ \alpha(\sin x)^{2p},\beta(\sin x)^{2v}\right]$$

$$\times I_{A,C:[B',D'];\dots;(B^{(n)}.D^{(n)})}^{0,\lambda:(\mu',v');\dots;(\mu^{(n)}.v^{(n)})}\left[\begin{matrix}[(a):\theta',\dots;\ \theta^{(n)}]:[(b'):\emptyset'];\dots;\\(c):\psi',\dots,\psi^{(n)}:[(d'):\delta'];\dots;\end{matrix}\right.$$

$$\left.\begin{matrix}\left[\left(b^{(n)}\right):\phi^{(n)}\right];\\\left[\left(b^{(n)}\right):\delta^{(n)}\right];\end{matrix}z_1(\sin x)^{2\sigma_1},\dots,z_n(\sin x)^{2\sigma_n}\right]dx$$

$$=\frac{\pi\,e^{imx/2}}{2^{w'-1}}\sum_{r,t=0}^{\infty}\in\frac{\left((\alpha/4^{\rho})\right)^r}{r!}\ \frac{\left((\beta/4^{\gamma})\right)^t}{t!}\times I_{A+1,c+2:[B',D'];\dots;(B^{(n)},.)}^{0,\lambda+1:(\mu',v');\dots;(\mu^{(n)}.v^{(n)})}\left[\begin{matrix}[(a):\theta'\dots;\ \theta^{(n)}],\\ [(c):\psi',\dots,\psi^{(n)}],\end{matrix}\right.$$

$$[1-\omega'-2pr-2\gamma t:2\sigma_1,\dots,2\sigma_n:\phi'];\dots;\left[\left(b^{(n)}\right):\left(\phi^{(n)}\right)\right];$$

$$\left.\left[\frac{1-\omega'-2_{\rho}r-2\gamma t\pm m}{2};\sigma_1,\dots,\sigma_n\right]:[(d'):\delta'];\dots{}'^{[(d^{(n)}):\delta^{(n)}]};\frac{z_1}{4^{\sigma_1}},\dots,\frac{z_n}{4^{\sigma_n}}\right] \tag{2.1}$$

Provide that $|\arg z_i| < \pi/2\ \Delta_i$,

$$\Delta_i=\sum_{j=1}^{\lambda}\theta_j^{(i)}-\sum_{j=\lambda+1}^{A}\theta_j^{(i)}+\sum_{j=1}^{v^{(i)}}\phi_j^{(i)}-\sum_{j=v^{(i)}+1}^{\beta^{(i)}}\phi_j^{(i)}-\sum_{j=1}^{C}\psi_j^{(i)}+\sum_{j=1}^{\mu^{(i)}}\delta_j^{(i)}-\sum_{j=\mu^{(i)}+1}^{D^{(i)}}\delta_j^{(i)}>0, i=1,\dots,n.$$

and $Re(\omega')>0;\alpha,\beta,\rho,\gamma,\sigma_1,z_i$ are positive integers, i =1,…, n. Now making an appeal to (1.1), we derive a multiple integral

$$\int_0^{\pi}\dots\int_0^{\pi}(\sin x_1)^{\omega_1-1}\dots(\sin x_1)^{\omega_n-1}e^{i}(m_1x_1+\dots+m_nx_n)$$

$$. F_{G_1:H_1:H_1'}^{E_1:F_1:F_1':}\left[\begin{matrix}(e_1):(f_1):(f_1');\\(g_1):(h_1):(h_1');\end{matrix}\alpha_1(\sin x_1)^{2\rho_1}, \beta_1(\sin x_1)^{2\gamma_1}\right]$$

$$\ldots \times F_{G_n:H_n:H_n'}^{E_n:F_n:F_n':}\left[\begin{matrix}(e_n):(f_n):(f_n');\\(g_n):(h_n):(h_n');\end{matrix}\alpha_n(\sin x_n)^{2\rho_n}, \beta_n(\sin x_n)^{2\gamma_n}\right]$$

$$\times I_{A,C:[B',D'];\ldots;(B^{(n)}.D^{(n)})}^{0,\lambda:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)})}\left[\begin{matrix}[(a):\theta'\ldots;\ \theta^{(n)}]:[(b'):\phi'];\ldots;\ldots;[b^{(n)}:\phi^{(n)}];z_1(\sin x_1)^{2\sigma_1{}^{(n)}}\\ \vdots \\ [(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;\ \ldots;(d^{(n)}:\delta^{(n)});z_1(\sin x_1)^{2\sigma_n{}^{(n)}}\end{matrix}\right]dx_1\ldots dx_n$$

$$= \frac{\pi^n\, e^{i(m_1+\cdots+m_n)\pi/2}}{2^{(\omega_1+\cdots+\omega_n)-n}} \sum_{r_1,t_1=0}^{\infty}\ldots\sum_{r_n,t_n=0}^{\infty}\ (\epsilon_1\ldots\epsilon_n)$$

$$\frac{\left(\frac{\alpha_1}{4^{\rho_1}}\right)^{r_1}}{r_1!}\frac{\left(\frac{\beta_1}{4^{r_1}}\right)^{t_1}}{t_1!}\frac{\left(\frac{\alpha_n}{4^{r_n}}\right)^{t_n}}{r_n!}\frac{\left(\frac{\beta_n}{4^{r_n}}\right)^{t_n}}{t_n!}$$

$$. I_{A+n,C+2n:[B',D'];\ldots;(B^{(n)}.D^{(n)})}^{0,\lambda+n:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)})}\left[\begin{matrix}[(a):\theta'\ldots;\ \theta^{(n)}],[1-\omega_1-2p_1r_1-2\gamma_1t_1:2\sigma_1',\ldots,2\sigma_n'],\ldots,\\ [(c):\psi';\ \ldots,\psi^{(n)}]\left[\frac{1-\omega_1-2p_1r_1-2\gamma_1t_1\pm m_n}{2}:\sigma_1',\ldots,\sigma_n'\right],\ldots,\end{matrix}\right.$$

$$\left[1-\omega_n-2p_nr_n-2\gamma_nt_n:2\sigma_n^{(n)},\ldots,2\sigma_n^{(n)}\right]:[(b'):\phi'];$$
$$\left[\frac{1-\omega_n-2p_nr_n-2\gamma_nt_n\pm m_n}{2}:\sigma_1^{(n)},\ldots,\sigma_n^{(n)}\right]:[(d')\delta'];$$

$$\left.\begin{matrix}\ldots;[(b^{(b)}):\phi^{(n)}];\\ \ldots;[(d^{(n)})\delta^{(n)}];\end{matrix}\frac{z_1}{4^{(\sigma_1'+\cdots+\sigma_1^{(n)})}},\ldots,\frac{z_n}{4^{(\sigma_n'+\cdots+\sigma_n^{(n)})}}\right] \tag{2.2}$$

provided that all conditions of (1.1) are satisfied and $Re\ (\omega_i) > 0; \sigma_1', \ldots, \sigma_1^{(n)}, \sigma_n', \ldots, +\sigma_n^{(n)}, \alpha_i, \beta_i, \rho_i, \gamma_i, z_i$ are positive integers $(i = 1, \ldots, n)$

3. EXPONENTIAL FOURIER SERIES:

Let

$$f(x) = (\sin x)^{\omega'-}. F_{G:H:H'}^{E:F:F'}\left[\begin{matrix}(e):(f):(f');\alpha(\sin x_1)^{2\rho}\\(g):(h):(h');\alpha_1(\sin x_1)^{2\gamma}\end{matrix}\right]$$

$$\times I_{A,C:[B',D'];\ldots;(B^{(n)}.D^{(b)})}^{0,\lambda:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)})}\left[\begin{matrix}[(a):\theta'\ldots;\ \theta^{(n)}]:[(b'):\phi'];\ldots;\\ [(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}\ldots;[b^{(n)}:\phi^{(n)}];\\ \ldots;[(d^{(n)}:\delta^{(n)});\end{matrix}\right] z_1(\sin x_1)^{2\sigma_1},\ldots,z_n(\sin x_1)^{2\sigma_n}$$

$$= \sum_{\rho=-\infty}^{\infty} A_\rho e^{-ipx} \tag{3.1}$$

which is valid due to $f(x)$ is continuous and of bounded variation with interval $(0, \pi)$.

Now, multiplying by e^{ipx} both series in (3.1) and integrating it with respect 0 to π from 0 to x, and then making an appeal to (1.2) and (2.1), We get

$$A_\rho = \frac{e^{ipx/2}}{2^{\omega'-1}} \sum_{r,t=0}^{\infty} \in \frac{(\alpha/4^\rho)^r}{r!} \frac{(\beta/4^r)^t}{t!} \times I^{0,\lambda+1:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)})}_{A+1,C+2:[B',D'];\ldots;(B^{(n)}.D^{(n)})} \left[\begin{matrix} [(a):\theta' \ldots;\ \theta^{(n)}] \\ [(c):\psi' \ \ldots, \psi^{(n)}] \end{matrix} \right.$$

$$[1-\omega' \ -2pr-2yt: 2\sigma_1, \ldots, 2\sigma_n]: [(b'):\phi']; \ldots; [(b^{(n)}):\phi^{(n)}];$$

$$\left[\frac{1-\omega' \ -2pr-2yt \pm m}{2}: \sigma_1, \ldots, \sigma_n\right]: [(d'):\delta']; \ldots; [(d^{(n)}):\delta']$$

$$\left. \frac{z_1}{4^{\sigma_1}}, \ldots, \frac{z_n}{4^{\sigma_n}} \right]. \tag{3.2}$$

An appeal to (3.1) and (3.2) gives the required exponential Fourier series

Integrating both sides with respect to x from 0 to x, we get

$$(2\sin x)^{\omega'-1} F^{E:F:F'}_{G:H:H'} \left[\begin{matrix} (e):(f):(f');\alpha(\sin x_1)^{2\rho} \\ (g):(h):(h');\beta(\sin x_1)^{2y}, \end{matrix} \right]$$

$$\times I^{0,\lambda:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)})}_{A,C:[B',D'];\ldots;(B^{(n)}.D^{(n)})} \left[\begin{matrix} [(a):\theta' \ldots;\ \theta^{(n)}]: [(b'):\phi']; \ldots; \ldots; [b^{(n)}:\phi^{(n)}]; \\ [(c):\psi', \ldots, \psi^{(n)}]: [(d'):\delta']; \ldots;\ \ldots; (d^{(n)}:\delta^{(n)}); \end{matrix} z_1(\sin x)^{2\sigma_1}, \ldots, z_n(\sin x)^{2\sigma_n} \right]$$

$$= \sum_{\rho=-\infty}^{\infty} \sum_{r,t=0}^{\infty} e^{ip(\pi/2-x)}.\in. \frac{(\alpha/4^\rho)^r}{r!} \frac{(\beta/4^r)^t}{t!} \times I^{0,\lambda+1:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)})}_{A+1,C+2:[B',D'];\ldots;(B^{(n)}.D^{(n)})}$$

$$\left[\begin{matrix} [(a):\theta', \ldots;\ \theta^{(n)}][1-\omega' \ -2pr-2yt: 2\sigma_1, \ldots, 2\sigma_n], : [(b'):\phi']; \ldots; [(b^{(n)}):\phi^{(n)}]; \\ [(c):\psi', \ldots, \psi^{(n)}] \left[\frac{1-\omega'-2pr-2yt \pm m}{2}: \sigma_1, \ldots, \sigma_n\right]: [(d'):\delta']; \ldots; [(d^{(n)}):\delta^{(n)}]; \end{matrix} \right.$$

$$\left. \frac{z_1}{4^{\sigma_1}}, \ldots, \frac{z_n}{4^{\sigma_n}} \right]. \tag{3.3}$$

4. Cosine Fourier Series:

Let

$$f(x) = (\sin x)^{\omega'-1} . F^{E:F:F'}_{G:H:H'} \left[\begin{matrix} (e):(f):(f');\alpha(\sin x_1)^{2\rho} \\ (g):(h):(h');\alpha_1(\sin x_1)^{2\gamma} \end{matrix} \right]$$

$$\times I^{0,\lambda:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)})}_{A,C:[B',D'];\ldots;(B^{(n)}.D^{(b)})} \left[\begin{matrix} [(a):\theta' \ldots;\ \theta^{(n)}]: [(b'):\phi']; \ldots; \ldots; [b^{(n)}:\phi^{(n)}]; \\ [(c):\psi', \ldots, \psi^{(n)}]: [(d'):\delta']; \ldots; \ldots; [(d^{(n)}:\delta^{(n)}); \end{matrix} \right] z_1(\sin x_1)^{2\sigma_1}, \ldots, z_n(\sin x_1)^{2\sigma_n}$$

$$= B_{0/2} + \sum_{\rho=1}^{\infty} B_\rho \cos px \tag{4.1}$$

Integrating both side with respect to x from π, we get

$$B_{0/2} = \frac{1}{\sqrt{\pi}} \sum_{r,t=0}^{\infty} \in \frac{(\alpha)^r}{r!} \frac{(\beta)^t}{t!}$$

$$I^{0,\lambda+1:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)})}_{A+1,C+r:[B',D'];\ldots;(B^{(n)}.D^{(n)})}\left[\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]\\ [(c):\psi',\ \ldots,\psi^{(n)}]\end{matrix}\right.$$

$$\left.\begin{matrix}\left[\frac{2-\omega'}{2}-pr-yt:\sigma_1,\ldots,\sigma_n\right],:[(b'):\phi'];\ldots;\left[\left(b^{(n)}\right):\phi^{(n)}\right];\\ \left[\frac{1-\omega'}{2}-pr-yt:\sigma_1,\ldots,\sigma_n\right]:[(d'):\delta'];\ldots;\left[\left(d^{(n)}\right):\delta^{(n)}\right];\end{matrix}\ z_1,\ldots,z_n\right] \quad (4.2)$$

Now, multiplying by both sides in (4.1) and integrating it the respect to x from 0 to x, and finally, making an appeal to (1.2), (1.3) and (2.1), we derive

$$B_p=\frac{e^{ipx/2}}{2-\omega'}\sum_{r,t=0}^{\infty}\in\frac{(\alpha)^r}{r!}\ \frac{(\beta)^t}{t!}$$

$$I^{0,\lambda+1:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)})}_{A+1,C+2:[B',D'];\ldots;(B^{(n)}.D^{(n)})}\left[\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}],\\ [(c):\psi',\ \ldots,\psi^{(n)}],\end{matrix}\right.$$

$$[1-\omega'-2pr-2yt:2\sigma_1,\ldots,2\sigma_n],:[(b'):\phi'];\ldots;\left[\left(b^{(n)}\right):\phi^{(n)}\right];$$

$$\left.\left[\frac{1-\omega'-2pr-2yt\pm m}{2}:\sigma_1,\ldots,\sigma_n\right]:[(d'):\delta'];\ldots;\left[\left(d^{(n)}\right):\delta^{(n)}\right];\frac{z_1}{4^{\sigma_1}},\ldots,\frac{z_n}{4^{\sigma_n}}\right] \qquad (4.3)$$

Using (4.2), (4.3), from (4.1) we get required cosine Fourier series

$$(\sin x)^{\omega'-1}.F^{E:F:F'}_{G:H:H'}\left[\begin{matrix}(e):(f):(f');\alpha(\sin x)^{2\rho}\\ (g):(h):(h');\alpha_1(\sin x)^{2\gamma}\end{matrix}\right]\times I^{0,\lambda:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)})}_{A,C:[B',D'];\ldots;(B^{(n)}.D^{(b)})}$$

$$\left[\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:[(b'):\phi'];\ldots;\ldots;[b^{(n)}:\phi^{(n)}];\\ [(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;\ \ldots;\left[\left(d^{(n)}:\delta^{(n)}\right);\right.\end{matrix}\right]z_1(\sin x_1)^{2\sigma_1},\ldots,z_n(\sin x_1)^{2\sigma_n}$$

$$=\frac{1}{\sqrt{\pi}}\sum_{r,t=0}^{\infty}\varepsilon\ \frac{(\alpha)^r}{r!}\ \frac{(\beta)^t}{t!}$$

$$I^{0,\lambda+1:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)})}_{A+1,C+1:[B',D'];\ldots;(B^{(n)}.D^{(n)})}\left[\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}],\\ [(c):\psi',\ \ldots,\psi^{(n)}],\end{matrix}\right.$$

$$\left.\begin{matrix}\left[\frac{2-\omega'}{2}-pr-yt:\sigma_1,\ldots,\sigma_n\right],:[(b'):\phi'];\ldots;\left[\left(b^{(n)}\right):\phi^{(n)}\right];\\ \left[\frac{1-\omega'}{2}-pr-yt:\sigma_1,\ldots,\sigma_n\right]:[(d'):\delta'];\ldots;\left[\left(d^{(n)}\right):\delta^{(n)}\right];\end{matrix}\ z_1,\ldots,z_n\right]$$

$$+\sum_{\rho=1}^{\infty}\sum_{r,t=0}^{\infty}\in e^{ipx/2}\cos px\ \frac{(\alpha/4^{\rho})^r}{r!}\ \frac{(\beta/4^{r})^t}{t!}$$

$$\frac{1}{2^{\omega'-2}}I^{0,\lambda+1:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)})}_{A+1,C+2:[B',D'];\ldots;(B^{(n)}.D^{(n)})}\left[\begin{matrix}[(a):\theta',\ldots;\ \theta^{(n)}],\\ [(c):\psi',\ \ldots,\psi^{(n)}],\end{matrix}\right.$$

$$[1-\omega'-2pr-2yt:2\sigma_1,\ldots,2\sigma_n]:[(b'):\phi'];\ldots;\left[\left(b^{(n)}\right):\phi^{(n)}\right];$$

$$\left[\frac{1-\omega'-2pr-2yt\pm m}{2}:\sigma_1,\ldots,\sigma_n\right]:[(d'):\delta'];\ldots;\left[\left(d^{(n)}\right):\delta^{(n)}\right]$$

$$\left.\frac{z_1}{4^{\sigma_1}},\ldots,\frac{z_n}{4^{\sigma_n}}\right]. \qquad (4.4)$$

5. Sine Fourier Series:

Let

$$f(x) = (\sin x)^{\omega'-1} . F^{E:F:F'}_{G:H:H'}\left[\begin{matrix}(e):(f):(f');\alpha(\sin x_1)^{2\rho}\\(g):(h):(h');\alpha_1(\sin x_1)^{2\gamma}\end{matrix}\right] \times I^{0,\lambda:(\mu',v');\dots;(\mu^{(n)}.v^{(n)})}_{A,C:[B',D'];\dots;(B^{(n)}.D^{(b)})}$$

$$\left[\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:[(b'):\phi'];..;\dots;[b^{(n)}:\phi^{(n)}]\\ [(c):\psi',\dots,\psi^{(n)}]:[(d'):\delta'];..;\dots;[(d^{(n)}:\delta^{(n)});]\end{matrix} z_1(\sin x_1)^{2\sigma_1},\dots,z_n(\sin x_1)^{2\sigma_n}\right.$$

$$= \sum_{\rho=1}^{\infty} C_\rho \sin px. \tag{5.1}$$

Multiplying by e^{imx} both sides in (5.1) and then integrating it with respect to x from 0 to x, and making an appeal to (1.4) and (6.1), we obtain

$$C_\rho = \frac{e^{ipx/2}}{i\,2^{\omega'-2}} \sum_{r,t=0}^{\infty} \in \frac{(\alpha/4^\rho)^r}{r!}\frac{(\beta/4^\gamma)^t}{t!} \times I^{0,\lambda+1:(\mu',v');\dots;(\mu^{(n)}.v^{(n)})}_{A+1,C+2:[B',D'];\dots;(B^{(n)}.D^{(n)})}$$

$$\left[\begin{matrix}[(a):\theta',\dots;\ \theta^{(n)}][1-\omega'-2pr-2yt:2\sigma_1,\dots,2\sigma_n]:[(b'):\phi'];..;[(b^{(n)}):\phi^{(n)}];\\ [(c):\psi',\ \dots,\psi^{(n)}]\left[\frac{1-\omega'-2pr-2yt\pm m}{2}:\sigma_1,\dots,\sigma_n\right]:[(d'):\delta'];\dots;[(d^{(n)}):\delta']\end{matrix}\ \frac{z_1}{4^{\sigma_1}},\dots,\frac{z_n}{4^{\sigma_n}}\right] \tag{5.2}$$

Now making an appeal to (5.1) we get required sine Fourier series

$$(\sin x)^{\omega'-1} . F^{E:F:F'}_{G:H:H'}\left[\begin{matrix}(e):(f):(f');\alpha(\sin x)^{2\rho}\\(g):(h):(h');\beta_1(\sin x)^{2\gamma}\end{matrix}\right] \times I^{0,\lambda:(\mu',v');\dots;(\mu^{(n)}.v^{(n)})}_{A,C:[B',D'];\dots;(B^{(n)}.D^{(b)})}$$

$$\left[\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:[(b'):\phi'];..;\dots;[b^{(n)}:\phi^{(n)}];]\\ [(c):\psi',\dots,\psi^{(n)}]:[(d'):\delta'];..;\ \dots;[(d^{(n)}:\delta^{(n)});]\end{matrix} z_1(\sin x_1)^{2\sigma_1},\dots,z_n(\sin x_1)^{2\sigma_n}\right]$$

$$= \sum_{\rho=-\infty}^{\infty}\sum_{r,t=0}^{\infty} \frac{2\in e^{ipx/2}}{i} \sin px \frac{(\alpha/4^\rho)^r}{r!}\frac{(\beta/4^\gamma)^t}{t!}$$

$$.I^{0,\lambda+1:(\mu',v');\dots;(\mu^{(n)}.v^{(n)})}_{A+1,C+2:[B',D'];\dots;(B^{(n)}.D^{(n)})}\left[\begin{matrix}[(a):\theta;\dots;\ \theta^{(n)}],\\ [(c):\psi;\ \dots,\psi^{(n)}],\end{matrix}\right.$$

$$[1-\omega'-2pr-2yt:2\sigma_1,\dots,2\sigma_n]:[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}];$$

$$\left[\frac{1-\omega'-2pr-2yt\pm m}{2}:\sigma_1,\dots,\sigma_n\right]:[(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}]$$

$$\left.\frac{z_1}{4^{\sigma_1}},\dots,\frac{z_n}{4^{\sigma_n}}\right]. \tag{5.3}$$

6. Multiple Exponential Fourier Series:

Let

$$f(x_1,\dots,x_n) = (\sin x_1)^{\omega_1-1}\dots(\sin x_n)^{\omega_n-1} F^{E:F:F'}_{G:H:H'}\left[\begin{matrix}(e):(f_1):(f_1{}');\alpha_1(\sin x_1)^{2\rho_1}\\(g_1):(h_1):(h_n');\beta_n(\sin x_n)^{2\gamma_1}\end{matrix}\right]$$

$$\dots F^{E_n:F_n:F_n'}_{G_n:H_n:H_n'}\left[\begin{matrix}(e_n):(f_n):(f_n');\alpha_n(\sin x_n)^{2\rho_n}\\(g_n):(h_n):(h_n');\beta_n(\sin x_n)^{2\gamma_n}\end{matrix}\right], \times I^{0,\lambda:(\mu',v');\dots;(\mu^{(n)}.v^{(n)})}_{A,C:[B',D'];\dots;(B^{(n)}.D^{(b)})}$$

$$\left[\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:[(b'):\phi'];\dots;\dots;[b^{(n)}:\phi^{(n)}];z_1(\sin x_1)^{2\sigma_1\dots(\sin x_1)^{2\sigma_1^{(n)}}}\\ \vdots \\ [(c):\psi',\dots,\psi^{(n)}]:[(d'):\delta'];\dots;\ \dots;(d^{(n)}:\delta^{(n)});z_n(\sin x_1)^{2\sigma_1{}'}\dots(\sin x_n)^{2\sigma_1^{(n)}}\end{matrix}\right]$$

$$= \sum_{\rho_1=-\infty}^{\infty}\dots\sum_{\rho_{n-1}=-\infty}^{\infty} A_{p_1\dots p_n}.\ e^{-i(p_1x_1+\dots+p_nx_1)} \qquad (6.2)$$

Equation (6.1) is valid, since $f(x_1,\dots,x_n)$ is continuous and of bounded variation in the open interval $(0,\infty)$. In the series (6.1), to calculate $A_{p_1\dots p_n}$ we fix $x_1,\dots,x_{n-1}$, so that

$$\sum_{\rho_1=-\infty}^{\infty}\dots\sum_{\rho_{n-1}=-\infty}^{\infty} A_{p_1\dots p_n}.\ e^{-i(p_1x_1+\dots+p_1x_1)}$$

Depends only on $\rho_{n.}$

Furthermore, it must be the coefficient of Fourier exponential series in x_n of $f\ (x_1,\dots,x_n)$ over $0<x_n<\pi$.

Now multiplying by $e^{im_nx_n}$ both sides in (6.1) and integrating with respect to x_n from 0 to π, we get

$$(\sin x_1)^{\omega_1-1}\dots(\sin x_{n-1})^{\omega_{n-1}-1} F^{E_1:F_1:F_1'}_{G_1:H_1:H_1'}\left[\begin{matrix}(e_1):(f_1):(f_1');\alpha_1(\sin x_1)^{2\rho_1}\\(g_1):(h_1):(h_1');\beta_1(\sin x_1)^{2\gamma_1}\end{matrix}\right]$$

$$\dots F^{E_{n-1}:F_{n-1}:F_{n-1}'}_{G_{n-1}:H_{n-1}:H_{n-1}'}\left[\begin{matrix}(e_{n-1}):(f_{n-1}):(f_{n-1}');\alpha_n(\sin x_n)^{2\rho_{n-1}}\\(g_{n-1}):(h_{n-1}):(h_{n-1}');\beta_{n-1}(\sin x_{n-1})^{2\gamma_{n-1}}\end{matrix}\right]$$

$$\times\int_0^{\pi}(\sin x_{n-1})^{\omega_{n-1}} e^{im_nx_n} F^{E_n:F_{rp}:F_n'}_{G_n:H_{rp}:H_n'}\left[\begin{matrix}(e_n):(f_n):(f_n');\alpha_n(\sin x_n)^{2\rho_n}\\(g_1):(h_1):(h_1');\beta_1(\sin x_1)^{2\gamma_n}\end{matrix}\right]$$

$$\times I^{0,\lambda:(\mu',v');\dots;(\mu^{(n)}.v^{(n)})}_{A,C:[B',D'];\dots;(B^{(n)}.D^{(b)})}\left[\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:[(b'):\phi'];\dots;\\ [(c):\psi',\dots,\psi^{(n)}]:[(d'):\delta'];\dots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(b^{(n)}):\phi^{(n)}];z_1(\sin x_1)^{2\sigma_n},\dots,(\sin x_n)^{2\sigma_1^{(n)}}\\ \vdots \\ [(d^{(n)}):\delta^{(n)}];z_n(\sin x_1)^{2\sigma_n},\dots,(\sin x_n)^{2\sigma_1^{(n)}}\end{matrix}\right]dx_n$$

$$= \sum_{\rho_1=-\infty}^{\infty}\dots\sum_{\rho_{n-1}=-\infty}^{\infty} A_{p_1\dots p_n}.\ e^{-i(p_1x_1+\dots+p_{n-1}x_{n-1})}$$

$$+\sum_{\rho_n=-\infty}^{\infty}\int_0^{\pi} e^{i(m_n-p_n)x_n}\ dx_1. \tag{6.2}$$

Using (1.2) and (2.1), from (6.2), repeatedly, we get

$$A_{p_1\ldots p_n=\sum_{r_1,t_1=0}^{\infty}\cdots\sum_{r_1,t_1=0}^{\infty}}\ \frac{e^{i(p_1+\cdots+p_n)\pi/2}}{2(\omega_1+\cdots+\omega_n)^{-n}}(\varepsilon_1\ldots\varepsilon_n)$$

$$.\frac{\left(\frac{\alpha_1}{4^{\rho_1}}\right)^{r_1}}{r_1!}\ \frac{\left(\frac{\beta_1}{4^{\gamma_1}}\right)^{t_1}}{t_1!}\cdots\frac{\left(\frac{\alpha_n}{4^{\rho_n}}\right)^{r_1}}{r_n!}\ \frac{\left(\frac{\beta_n}{4^{\gamma_n}}\right)^{t_n}}{t_n!}$$

$$\times I^{0,\lambda+n:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)});\ldots;}_{A+n,C+2n:[B',D'];\ldots;\,(B^{(n)}.D^{(b)})}\left[\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]\\ [(c):\psi',\ldots,\psi^{(n)}]\end{matrix}\right.$$

$$[1-\omega_1\ -2p_1r_1-2y_1t_1:2\sigma_1',\ldots,2\sigma_n'],\ldots,$$

$$\left[\frac{1-\omega_1\ -2p_1r_1-2y_1t_1\pm m_1}{2}:\sigma_1',\ldots,\sigma_n'\right.$$

$$[1-\omega_n\ -2p_nr_n-2y_nt_n:2\sigma_1',\ldots,2\sigma_n']:[(b'):\phi'];$$

$$\left[\frac{1-\omega_n\ -2p_nr_n-2y_nt_n\pm m_n}{2}:\sigma_1^{(n)},\ldots,\sigma_n^{(n)}\right]:[(d'):\delta'];$$

$$\left.\begin{matrix}\ldots;[(b^{(n)}):\phi^{(n)}];\\ \ldots;\left[\left((d^{(n)}):\delta^{(n)}\right)\right];\frac{z_1}{4^{\left(v_1'+\cdots+\sigma_1^{(n)}\right)}},\ldots,\frac{z_n}{4^{\left(\sigma_1'+\cdots+\sigma_1^{(n)}\right)}}\end{matrix}\right]. \tag{6.3}$$

Using (6.3), in (6.1) we get required multiple exponential Fourier series

$$(\sin x_1)^{\omega_1-1}\ldots(\sin x_n)^{\omega_{n-1}}\ F^{E_1:F_1:F_1'}_{G_1:H_1:H_1'}\left[\begin{matrix}(e_1):(f_1):(f_1');\alpha_1(\sin x_1)^{2\rho_1}\\ (g_1):(h_1):(h_1');\beta_1(\sin x_1)^{2\gamma_1}\end{matrix}\right]$$

$$\ldots F^{E_n:F_n:F_n'}_{G_n:H_n:H_n'}\left[\begin{matrix}(e_n):(f_n):(f_n');\alpha_n(\sin x_n)^{2\rho_n}\\ (g_n):(h_n):(h_n');\beta_n(\sin x_n)^{2\gamma_n}\end{matrix}\right]$$

$$\times I^{0,\lambda:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)})}_{A,C:[B',D'];\ldots;\,(B^{(n)}.D^{(b)})}\left[\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:[(b'):\phi'];\ldots;\\ [(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(b^{(n)}):\phi^{(n)}];z_1(\sin x_1)^{2\sigma_1'},\ldots,(\sin x_n)^{2\sigma_1^{(n)}}\\ \vdots\\ [(d^{(n)}):\delta^{(n)}];z_n(\sin x_1)^{2\sigma_n'},\ldots,(\sin x_n)^{2\sigma_1^{(n)}}\end{matrix}\right]dx_n$$

$$=\sum_{\rho_1,\ldots,\rho_n=-\infty}^{\infty}\cdots\sum_{r_1,\ldots,r_n.t_1,\ldots,t_n=0}^{\infty}\frac{\epsilon_1\ldots\epsilon_n}{2(\omega_1+\cdots+\omega_n)-n}e^{-i(p_1x_1+\cdots+p_nx_n)}$$

$$.e^{i(p_1+\cdots+p_n)\pi/2}\frac{\left(\frac{\alpha_1}{4^{\rho_1}}\right)^{r_1}}{r_1!}\ \frac{\left(\frac{\beta_1}{4^{\gamma_1}}\right)^{t_1}}{t_1!}\cdots\frac{\left(\frac{\alpha_n}{4^{\rho_n}}\right)^{r_n}}{r_n!}\ \frac{\left(\frac{\beta_n}{4^{\gamma_n}}\right)^{t_n}}{t_n!}$$

$$\times I^{0,\lambda+n:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)});\ldots;}_{A+n,C+2n:[B',D'];\ldots;\,(B^{(n)}.D^{(b)})}\left[\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]\\ [(c):\psi',\ldots,\psi^{(n)}]\end{matrix}\right.$$

$$\left[1-\omega_1-2p_1r_1-2y_1t_1 : 2\sigma_1', \dots, 2\sigma_n'\right], \dots,$$

$$\left[\frac{1-\omega_1-2p_1r_1-2y_1t_1 \pm m_1}{2} : \sigma_1', \dots, \sigma_n'\right], \dots,$$

$$\left[1-\omega_n-2p_nr_n-2y_nt_n : 2\sigma_1^{(n)}, \dots, 2\sigma_n^{(n)}\right] : [(b') : \phi'];$$

$$\left[\frac{1-\omega_n-2p_nr_n-2y_nt_n \pm m_n}{2} : \sigma_1^{(n)}, \dots, \sigma_n^{(n)}\right] : [(d') : \delta'];$$

$$\left.\begin{matrix} \dots;\left[(b^{(n)}) : \phi^{(n)}\right]; \\ \dots;\left[(d^{(n)}) : \delta^{(n)}\right]; \end{matrix} \frac{z_1}{4^{\left(\sigma_1'+\dots+\sigma_1^{(n)}\right)}}, \dots, \frac{z_n}{4^{\left(\sigma_1'+\dots+\sigma_n^{(n)}\right)}}\right] \tag{6.4}$$

7. Particular Cases:

Putting $\beta_1, \dots, \beta_n = 0$ in (2.2), we get

L.H.S. of (7.1)

$$\int_0^{\pi} \dots \int_0^{\pi} (\sin x_1)^{\omega_1 - 1} \dots (\sin x_n)^{\omega_n - 2} e^{i(m_1x_1+\dots+m_nx_n)}$$

$$\times\, {}_{E_1+F_1}F_{G_1+H_1}\left[\begin{matrix}(e),(f_1); \\ (g_1),(h_1)\end{matrix} \alpha_1 (\sin x_1)^{2\rho_1}\right]$$

$$\dots\, {}_{E_n+F_n}F_{G_n+H_n}\left[\begin{matrix}(e_n),(f_n); \\ (g_n),(h_n)\end{matrix} \alpha_n (\sin x_n)^{2\rho_n}\right]$$

$$\times I^{0,\lambda:(\mu',v');\dots;(\mu^{(n)}.v^{(n)})}_{A,C:[B',D'];\dots;(B^{(n)}.D^{(b)})}\left[\begin{matrix}\left[(a):\theta',\dots,\theta^{(n)}\right]:[(b'):\phi'];\dots; \\ \left[(c):\psi',\dots,\psi^{(n)}\right]:[(d'):\delta'];\dots;\end{matrix}\right.$$

$$\left.\begin{matrix}\left[(b^{(n)}):\phi^{(n)}\right]; z_1(\sin x_1)^{2\sigma_1'}, \dots, (\sin x_n)^{2\sigma_1^{(n)}} \\ \vdots \\ \left[(d^{(n)}):\delta^{(n)}\right]; z_n(\sin x_1)^{2\sigma_n'}, \dots, (\sin x_n)^{2\sigma_1^{(n)}}\end{matrix}\right] dx_1 \dots dx_n$$

$$\frac{(\pi)^n e^{i(m_1+\dots+m_n)\pi/2}}{2(\omega_1+\dots+\omega_n)^{-n}} \sum_{r_1,\dots,r_n=0}^{\infty} 0 \frac{\prod_{k_1=1}^{E_1}\left(e_{1_{k_1}}\right)_{r_1}\prod_{k_1=1}^{F_1}\left(f_{1_{k_1}}\right)_{r_1}}{\prod_{k_1=1}^{G_1}\left(g_{1_{k_1}}\right)_{r_1}\prod_{k_1=1}^{H_1}\left(h_{1_{k_1}}\right)_{r_1}}$$

$$\frac{\prod_{k_n=1}^{E_n}\left(e_{n_{k_n}}\right)_{r_n}\prod_{k_1=1}^{F_n}\left(f_{n_{k_n}}\right)_{r_n}}{\prod_{k_n=1}^{G_n}\left(g_{n_{k_n}}\right)_{r_n}\prod_{k_n=1}^{H_n}\left(h_{n_{k_n}}\right)_{r_n}} \frac{\left(\frac{\alpha_1}{4^{\rho_1}}\right)^{r_1}}{r_1!} \dots \frac{\left(\frac{\alpha_n}{4^{\rho_n}}\right)^{r_n}}{r_n!}$$

$$\times I^{0,\lambda+n:(\mu',v');\dots;(\mu^{(n)}.v^{(n)});\dots;}_{A+n,C+2n:[B',D'];\dots;(B^{(n)}.D^{(n)})}\left[\begin{matrix}\left[(a):\theta',\dots,\theta^{(n)}\right] \\ \left[(c):\psi',\dots,\psi^{(n)}\right]\end{matrix}\right.$$

$$\left[1-\omega_1-2p_1r_1 : 2\sigma_1', \dots, 2\sigma_n'\right], \dots,$$

$$\left[\frac{1-\omega_1-2p_1r_1 \pm m_1}{2} : \sigma_1', \dots, \sigma_n'\right], \dots,$$

$$\left[1-\omega_n-2p_nr_n: 2\sigma_1^{(n)},\ldots,2\sigma_n^{(n)}\right]:[(b'):\phi'];$$
$$\left[\frac{1-\omega_n-2p_nr_n\pm m_n}{2}:\sigma_1^{(n)},\ldots,\sigma_n^{(n)}\right]:[(d'):\delta'];$$
$$\left.\begin{matrix}\ldots;\left[\left(b^{(n)}\right):\phi^{(n)}\right];\\ \ldots;\left[\left(d^{(n)}\right):\delta^{(n)}\right];\end{matrix}\frac{z_1}{4^{\left(\sigma_1'+\cdots+\sigma_1^{(n)}\right)}},\ldots,\frac{z_n}{4^{\left(\sigma_1'+\cdots+\sigma_n^{(n)}\right)}}\right] \tag{7.1}$$

Further putting $\alpha_1,\ldots,\alpha_n=0$ in (7.1), we obtain

L.H.S. of (7.2)

$$\int_0^{\pi}\ldots\int_0^{\pi}(\sin x_1)^{\omega_1-1}\ldots(\sin x_n)^{\omega_n-1}e^{i(m_1x_1+\cdots+m_nx_n)}$$

$$\times I_{A,C:\,(B^n.D^n)}^{0,\lambda:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)})}\left[\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:[(b'):\phi'];\ldots;\\ [(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}\left[\left(b^{(n)}\right):\phi^{(n)}\right];z_1(\sin x_1)^{2\sigma_1'},\ldots,(\sin x_n)^{2\sigma_1^{(n)}}\\ \vdots\\ \left[\left(d^{(n)}\right):\delta^{(n)}\right];z_n(\sin x_1)^{2\sigma_n'},\ldots,(\sin x_n)^{2\sigma_1^{(n)}}\end{matrix}\right]dx_1\ldots dx_n$$

$$\frac{(\pi)^n e^{i(m_1+\cdots+m_n)\pi/2}}{2(\omega_1+\cdots+\omega_n)^{-n}}$$

$$I_{A+n,C+2n:[B',D'];\ldots;\,(B^{(n)}.D^{(n)})}^{0,\lambda+n:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)});\ldots;}\left[\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]\\ [(c):\psi',\ldots,\psi^{(n)}]\end{matrix}\right.$$

$$\left[1-\omega_1-:2\sigma_1',\ldots,2\sigma_n'\right],\ldots,\left[1-\omega_n-:2\sigma_1^{(n)},\ldots,2\sigma_n^{(n)}\right]$$

$$\left[\frac{1-\omega_n\pm m_1}{2}-:\sigma_1',\ldots,\sigma_n'\right],\ldots,\left[\frac{1-\omega_n\pm m_n}{2}-:\sigma_1^n,\ldots,\sigma_n^n\right]$$

$$\left.\begin{matrix}[(b'):\phi'];\ldots;\left[\left(b^{(n)}\right):\phi^{(n)}\right];\\ [(d'):\delta'];\ldots;\left[\left(d^{(n)}\right):\delta^{(n)}\right];\end{matrix}\frac{z_1}{4^{\left(\sigma_1'+\cdots+\sigma_1^{(n)}\right)}},\ldots,\frac{z_n}{4^{\left(\sigma_1'+\cdots+\sigma_n^{(n)}\right)}}\right].$$

Now putting $\alpha+\beta=0$ in (3.3), we establish

$$(\sin x_1)^{\omega_1-1}I_{A,C:\,[B',D'];\ldots;\,(B'.D')}^{0,\lambda:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)})}\left[\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:[(b'):\phi'];\ldots;\\ [(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[b^{(n)}:\phi^{(n)}];]\\ [(d^{(n)}:\delta^{(n)});]\end{matrix}z_1(\sin x_1)^{2\sigma_1},\ldots,z_n(\sin x_1)^{2\sigma_n}\right]$$

$$=\sum_{p=-\infty}^{\infty}\frac{e^{ip(\frac{\pi}{2}-x)}}{2^{\omega'-1}}I_{A+1,C+2:[B',D'];\ldots;\,(B^{(n)},D^{(n)})}^{0,\lambda+1:(\mu',v');\ldots;(\mu^{(n)}.v^{(n)});\ldots;}\left[\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}],\\ [(c):\psi',\ldots,\psi^{(n)}],\end{matrix}\right.$$

$$[1-\omega':2\sigma_1,\ldots,2\sigma_n:]:[(b'):\phi'];\ldots;\left[\left(b^{(n)}\right):\phi^{(n)}\right];$$

$$\left[\frac{1-\omega'-\pm\rho}{2};\sigma_1,\dots,\sigma_n\right]:[(d'):\delta'];\dots\left[\left(d^{(n)}\right):\delta^{(n)}\right];\frac{z_1}{4^{\sigma_1}},\dots,\frac{z_n}{4^{\sigma_n}}\right] \tag{7.3}$$

Letting $\rho = 2l$ as an integer, from (7.3) we establish

L.H.S. of (7.3)

$$= \frac{1}{\sqrt{\pi}} I^{0,\lambda+1:(\mu',v');\dots;(\mu^{(n)}.v^{(n)});\dots;}_{A+1,C+1:[B',D'];\dots;(B^{(n)},D^{(n)})} \left[\begin{matrix}[(a):\theta',\dots,\theta^{(n)}],\\ [(c):\psi',\dots,\psi^{(n)}],\end{matrix}\right.$$

$$\left.\begin{matrix}\left[\frac{2-\omega'}{2}:\sigma_1,\dots,\sigma_n\right]:[(b'):\phi'];\dots;\left[b^{(n)}:\phi^{(n)}\right];\\ \left[\frac{1-\omega'}{2}:\sigma_1,\dots,\sigma_n\right]:[(d'):\delta'];\dots;\left[d^{(n)}:\delta^{(n)}\right];\end{matrix}\ z_1,\dots,z_n\right]$$

$$+\frac{1}{2^{\omega'-2}}\sum_{p=1}^{\infty}\cos l\pi\cos 2lx\ I^{0,\lambda+1:(\mu',v');\dots;(\mu^{(n)}.v^{(n)});\dots;}_{A+1,C+2:[B',D'];\dots;(B^{(n)},D^{(n)})}$$

$$\left[\begin{matrix}[(a):\theta',\dots,\theta^{(n)}],[1-\omega'-2\sigma_1,\dots,2\sigma_n]:[(b'):\phi'];\dots;\left[\left(b^{(n)}\right):\phi^{(n)}\right];\\ \left[(c):\psi',\dots,\psi^{(n)}\right],\frac{1-\omega'\pm 2l}{2}:\sigma_1,\dots,\sigma_n\right]:[(d'):\delta'];\dots;\left[\left(d^{(n)}\right):\delta^{(n)}\right];\end{matrix}\right.$$

$$\left.\frac{z_1}{4^{\sigma_1}},\dots,\frac{z_n}{4^{\sigma_n}}\right] \tag{7.4}$$

Further letting $p = (2l+1)$ as l is an integer, from (7.3) we obtain

L.H.S. of (7.3) $= \frac{1}{2^{\omega'-2}}\sum_{p=1}^{\infty}\sin(2l+1)\pi/2.\sin(2l+1)x$

$$\times I^{0,\lambda+1:(\mu',v');\dots;(\mu^{(n)}.v^{(n)});\dots;}_{A+1,C+2:[B',D'];\dots;(B^{(n)},D^{(n)})}$$

$$\left[\begin{matrix}[(a):\theta',\dots,\theta^{(n)}],[1-\omega'-2\sigma_1,\dots,2\sigma_n]:[(b'):\phi'];\dots;\left[\left(b^{(n)}\right):\phi^{(n)}\right];\\ \left[(c):\psi',\dots,\psi^{(n)}\right],\frac{1-\omega'\pm(2l+1)}{2}:\sigma_1,\dots,\sigma_n\right]:[(d'):\delta'];\dots;\left[\left(d^{(n)}\right):\delta^{(n)}\right];\end{matrix}\right.$$

$$\left.\frac{z_1}{4^{\sigma_1}},\dots,\frac{z_n}{4^{\sigma_n}}\right] \tag{7.5}$$

Similarly, remaining particular cases can be evaluated by (4.4) and (5.3) applying the same techniques.

It is interesting to note that, after little simplifications, from (7.1), (7.2), (7.3) (7.4) and (7.5), we can be obtained as special cases of our results.

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