



The Analysis Of A Mathematical Modelling In Two Phase Blood Flow In Arterioles During Thalassemia

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Abstract: In this theoretical study, a mathematical model is developed to carry out a systematic analysis of flow behaviour in a two-dimensional vessel (modeled as artery) with a locally variable shaped constriction. An artificial artery, which containing a viscous incompressible fluid that representing the flowing blood can be treated as inflexible vessel. The shape of the stenosis in the arterial lumen is chosen to be symmetric as well as asymmetric about the middle cross section is perpendicular to the axis of the vessel. The constricted 1 is resolved into a straight el and the entire resulting aquations are solved hu numerical.

Keywords: Two-dimensional vessel, Artificial artery, Incompressible, Stenosed artery, Reynolds number, Resistance, asymmetric

1. INTRODUCTION

It is known that stenosis is a constricting of any tubulike structure in the body, including blood vessels, heart valves, vertebral canal and the GI tract. A survey of the existing literature on the experimental and theoretical studies of blood flow various parts of the arterial tree under normal as well as pathological conditions further indicates that in certain situations the Bingham plastic fluid model (a particular case of Herschel-Bulkley model) gives a better description of the rheological properties of blood [2, 16]. Blood vessel narrowing is one of the more common usages of this term. High cholesterol could conduced to the stenosis of an artery when it accumulates on the inner wall of the artery. This accumulation, referred to as atherosclerosis, can build up within the artery to the point where it reduces blood flow to organs of the body. When blood flow is reduced, nutrients and oxygen cannot travel to the tissues that need it. The presence of stenosis can lead to serious circulatory disorders. There is strong evidence that hydrodynamic factors such as resistance to flow, wall shear stress and apparent viscosity may play a vital role in the development and the progression of arterial stenosis. Young [18] described many problems in fluid mechanics of arterial stenosis, while Shukla et al. [12] studied the effect of stenosis on non-Newtonian flow of blood in an artery. Many researchers [15,17] feel that the hydrodynamic factors may be helpful in the diagnosis, treatment and fundamental understanding of many disorders. Verma et. al. [14] studied the shape of stenosis to blood flow through an artery with mild stenosis and they evaluated, for a given rate of flow, the wall shear stress increases immediately as the stenosis increases in size. Misra & Shit [3] investigated blood flow through arterial segment assuming blood as Hershal-Bulkley fluid. They obtained that the skin-friction and the resistance to flow is maximum at the throat of the stenosis and minimum at the end. Ali et. al. [8] analyzed the effect of an axially symmetric time dependent growth into the lumen of a tube for constant cross section through which a Newtonian fluid is steadily flowing .They investigated the structure of flow through arterial

model with one or two sinusoidal stenosis, assuming the arterial blood flow is quasi-steady. Shah & Siddiqui [10] studied the influence of peripheral layer viscosity on physiological characteristics of blood flow through stenosed artery using Power-law fluid model. It was observed that the resistance to flow increases as stenosis size and peripheral layer viscosity increases. They found that the peripheral layer viscosity of blood in diabetic patients is higher than in non-diabetic patients, resulting in higher resistance to blood flow. Thus diabetic patients with higher peripheral layer viscosity are more prone to high blood pressure. Therefore the resistance to blood flow in case of diabetic patients may be reduced by reducing viscosity of the plasma. A mathematical model of blood flow in porous vessel having double stenosis in the presence of an external magnetic field has been investigated by Sinha et al. [13] while Shit and Roy [11] put forward a mathematical analysis for the unsteady flow of blood through arteries having stenosis, in which blood was treated as a Newtonian viscous incompressible fluid. They also investigated that the wall shear stress decreases as stenosis shape parameter increases but in the case of increasing stenosis size, stenosis length and peripheral layer viscosity wall shear stress increases. Bali & Awasthi [9] analyzed the effect of external magnetic field on blood flow in stenotic artery. They presented that the resistance to flow increases with stenosis height and red cell, which depends on hematocrit value of blood, on the other side resistance to flow decreases with the increase in the value of Hartmann number. Mathur & Jain [7] studied the pulsatile flow of blood through stenosed arteries, including the effects of body acceleration and a magnetic field. Blood flow problems are more complicated than the fluid flow problems. Some researchers studied the power law fluid model of blood arguing that under certain conditions, blood behaves like a power law fluid. Casson [6] examined the validity of Casson model in studies pertaining to the flow characteristics of blood and reported that at low shear rates the yield stress for blood is non-zero. The properties arise from the fact that the vessel walls are formed of different substances such as elastic, collagen and smooth muscles with entirely different properties.

In present paper we have studied the effects of bell shaped stenosis on resistance to flow, wall shear stress and velocity profile in an artery by assuming the blood as power-law fluid.

Thalassemia is an inherited blood disorder. It causes the body to make less haemoglobin. There are two main types of thalassemia; alpha and beta, Beta thalassemia is affected by different genes of parents suffering from thalassemia. There are several types of beta thalassemia; major and minor, different people will have different symptoms, based on which type of major beta thalassemia (Cooley Anemia) is reduced the production of haemoglobin. Haemoglobin is the iron-containing protein in red blood cells and plasma that carries oxygen to cells throughout the body.

In people with major beta thalassemia (Cooley Anemia) low levels of haemoglobin lead to a lack of oxygen in many parts of the body. Affected individuals also have a shortage of red blood cells and plasma, which can cause pale skin, fussy, having a poor appetite, having many infections, belly swelling and slowed growth. People with major beta thalassemia are at an increased risk of developing abnormal blood clots,

Structure and the function of arterioles

An arteriole is a small-diameter blood vessel in the microcirculation that extends and branches out from an artery and leads to capillaries. Arterioles have muscular walls (usually only one to two layers of smooth muscle cells) and are the primary site of vascular resistance. The greatest change in blood pressure and velocity of blood flow and plasma occurs at the transition of arterioles to capillaries. This function is extremely important because it prevents the thin, one-layer capillaries from exploding upon pressure. The arterioles achieve this decrease in pressure, as they are the site with the highest resistance

Real Model

Frame of reference: - In this model we have chosen orthogonal curvilinear generalized three-dimensional co-ordinate system denoted by E called three-dimensional Euclidean space of the moving blood. All quantities related to blood flow written in tensorial form which is comparatively more realistic. Let P be a point in space with co-ordinate X^i with respect to axes OX^i , O as origin where $i = 1, 2, 3$. At time t , $V_k = V_k(X^i, t)$ be velocity of blood, $\rho = \rho(X^i, t)$.

thermodynamically pressure and $p = p(X^i, t)$ density, Since blood vessels are cylindrical the governing equations have to transform into cylindrical co-ordinate system ^[6].

Constitution of blood

Blood is bio fluid or fluid connective tissue. Blood consists of a suspension of cells in an aqueous solution called plasma which composed of about 90% water and 7% protein. There are about 95% are red blood cells or erythrocytes whose main function is to transport oxygen from lungs to all the cells of the body and removal of carbon dioxide formed by metabolic process in the body to lungs. About 45% of the blood volume in an average human is occupied by red blood cells. This fraction is known as the hematocrit of the remaining white blood cells or leucocytes constitute about one sixth or 1% of total and these play a important role in the body to infection and platelets form 5% of the total blood and they perform a function related to blood clotting [*]

Formulation

According to Sharman I.W. and Sherman V.G. blood is mixed fluid. There are two phases in the blood, one is plasma and other is blood cells. The blood cells are enclosed with a semi- permeable membrane whose density is greater than that of plasma. These blood cells are uniformly distributed in plasma.

Equation of continuity for two phase blood

According to Upadhyay V. The flow of blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells. Let the volume portion covered by blood cells in unit volume be X , where $X = \frac{H}{100}$ and H is hematocrit the volume percentage of blood cells. Then the volume portion covered by plasma will be $1 - X$. If the mass ratio of blood cells to plasma is r then

$$r = \frac{X_{p_c}}{(1 - X)P_p}$$

Where P_c and P_p are densities of blood cells and plasma respectively. The both phase the blood cells and plasma move with common velocity. Campbell and Pitcher have presented a model for this condition. Equation of continuity for two phases according to principle of conservation of mass defined by J.N., and Gupta R. C. As follows-

$$\frac{\partial X \rho_c}{\partial t} + (X \rho_p)_{,i} = 0$$

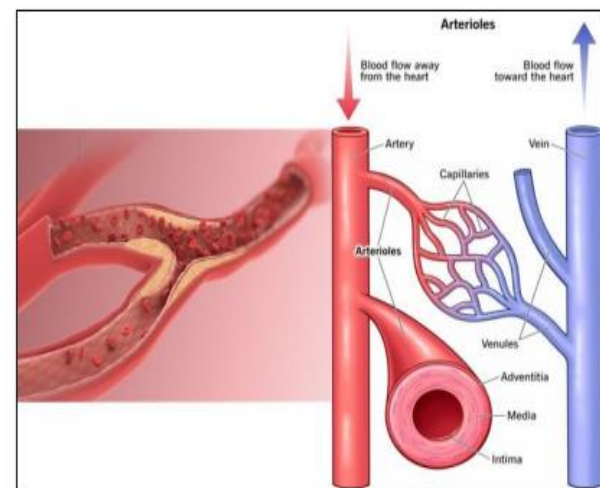


Fig 1: Blood Flow in Human Body ^[21]

II. FORMULATION OF THE PROBLEM AND ITS SOLUTION

Let us consider a bell shaped stenosis geometry given by [3]

$$R(z) = R_0 \left[1 - \frac{\delta}{R_0} \exp \left(\frac{-m^2 \varepsilon^2 z^2}{R_0^2} \right) \right] \quad (1)$$

Where R_0 stands for the radius of the arterial segment outside the stenosis, $R(z)$ is the radius of the arterial segment under consideration of a longitudinal distance

z from the left end of the segment, δ is the depth of the stenosis at the throat and m is a parametric constant, ε is characterizes the relative length of the constriction, defined as the ratio of the radius to half length of stenosis,

$$\text{i.e. } \varepsilon = \frac{R_0}{L_0}$$

Considering the stenosis geometry to be of the form

$$\frac{R(z)}{R_0} = [1 - ae^{-bz^2}] \quad (2)$$

With

$$a = \frac{\delta}{R_0} \text{ and } b = \frac{m^2 \varepsilon^2}{R_0^2}$$

Let us consider blood as a power law fluid, the constitutive equation for power law fluid is given as

Or

$$\tau = \mu e^n$$

$$\left(\frac{\tau}{2\mu} \right)^{\frac{1}{n}} = e = \left(\frac{1}{2} \frac{pr}{\mu} \right)^{\frac{1}{n}} = - \frac{du}{dr}$$

$$\mu = \left(\frac{P}{2\mu} \right)^{\frac{1}{n}} \frac{n}{n+1} r^{\frac{1}{n}+1} + c \quad (3)$$

Using boundary conditions

$$\tau \text{ is finite at } r = 0 \text{ (regularity condition)} \quad (4)$$

$$u = 0 \text{ at } r = R(z) \text{ (no-slip condition)} \quad (5)$$

$$u = \left(\frac{P}{2\mu} \right)^{\frac{1}{n}} \frac{n}{n+1} \left(R_0^{\frac{1}{n}+1} - r^{\frac{1}{n}+1} \right) \quad (6)$$

In the absence of the stenosis ($\delta = 1$)

$$u_p = \left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{n}{n+1} \left(R_0^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right) \quad (7)$$

Where the subscript p denotes the poiseuille flow

$$\frac{u}{u_p} = \frac{\left(R_0^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right)}{\left(R_0^{\frac{1}{n}+1} - r^{\frac{1}{n}+1}\right)} \quad (8)$$

Flow rate can be given as [4]

$$Q = \int_0^R u 2\pi r dr$$

$$Q = \pi \left(\frac{P}{2\mu}\right)^{\frac{1}{n}} \frac{R^{\frac{1}{n}+3}}{\frac{1}{n}+3} \quad (9)$$

$$P = \frac{2\mu}{R_0^{1+3n}} \left(\frac{Q}{\pi} \left(\frac{1}{n} + 3\right)\right)^n \quad (10)$$

Pressure drop across the length of the stenosis

$$\Delta P = \frac{2\mu}{R_0^{1+3n}} \left(\frac{Q}{\pi} \left(\frac{1}{n} + 3\right)\right)^n \int_{-z_0}^{z_0} \frac{dz}{\left(\frac{R}{R_0}\right)^{1+3n}} \quad (11)$$

In the absence of the stenosis ($\delta = 0$)

$$(\Delta P)_{\delta=0} = \frac{2\mu}{R_0^{1+3n}} \left(\frac{Q}{\pi} \left(\frac{1}{n} + 3\right)\right)^n z_0 \quad (12)$$

$$K = \frac{\Delta P}{(\Delta P)_{\delta=0}}$$

Or

$$K = \frac{1}{2z_0} \int_{-z_0}^{z_0} \frac{dz}{\left(\frac{R}{R_0}\right)^{1+3n}}$$

$$\begin{aligned} \Delta P = & \frac{4\mu}{R_0^{1+3n}} \left(\frac{Q}{\pi} \left(\frac{1}{n} + 3\right)\right)^n (2L - 2z_0) \\ & + \frac{2\mu}{R_0^{1+3n}} \left(\frac{Q}{\pi} \left(\frac{1}{n} + 3\right)\right)^n \int_{-z_0}^{z_0} \frac{dz}{\left(\frac{R}{R_0}\right)^{1+3n}} \end{aligned} \quad (14)$$

When there is no stenosis

$$(\Delta P)_{\delta=0} = \frac{4\mu}{R_0^{1+3n}} \left(\frac{Q}{n} \left(\frac{1}{n} + 3 \right) \right)^n L \quad (15)$$

$$K_1 = \frac{P}{(\Delta P)_{\delta=0}} = 1 - \frac{Z_0}{L} + \frac{1}{2L} \int_{-Z_0}^{Z_0} \frac{dz}{\left(\frac{R}{R_0} \right)^{1+3n}} \quad (16)$$

The shear stress on the surface of the stenosis is given by

$$\tau = \frac{1}{2} P(z) R(z)$$

$$\tau = \frac{1\mu}{R^{3n}} \left(\frac{Q}{\pi} \left(\frac{1}{n} + 3 \right) \right)^n \quad (17)$$

Also the ratio of shearing stress on with and without stenosis can be written as;

$$\tau = \frac{\tau}{\tau_p} = \left(\frac{R}{R_0} \right)^{3n} \quad (18)$$

II. RESULTS & CONCLUSION

In this paper, we have dealt with the effects of stenosis in an artery by considering the blood as power-law fluid. It has been concluded that the pressure drop and shear stress increases as the size of the stenosis increases for a given non-Newtonian model of the blood. The expression for velocity in Eqs. (8), for pressure drops across the length of the stenosis in Eqs. (13), for pressure drops along the length of the artery in Eqs. (16) and for shear stress in Eqs. (18) and displayed graphically. Figure (1) & (2) shows variation of pressure drop across the length of the stenosis w. r. t. stenosis height and the relative length of the constriction and highlights that the pressure drop increases with increases and decreases with $\frac{\delta}{R_0}$ increases. Eqs. (16) Depicts that the pressure drop along the length of the artery increases with $\frac{\delta}{R_0}$ and increases with $\frac{Z_0}{L}$ increases for fixed n , this is in conformity with the results obtained by Kapur [4]. Variation of wall shear stress with axial distance z , stenosis height and with ϵ are presented in fig. (2), fig. (3) & fig. (5) respectively. Form the figure 3, it can be clearly observed that shear stress increases from its approached magnitude (i.e. at $z = -2.5$) in the upstream of the throat with the axial distance and achieves its maximal at the throat of the stenosis and then decreases in the downstream and attains a lower magnitude at the end of the constriction profile (i.e. at $z=2.5$). Magnitude of wall shear stress in uniform tube is lower than its magnitude in stenosed artery; this is in conformity with the results obtained by Biswas and Chakraborty [1]. It can be seen from fig (4) that shear stress increases with the height of stenosis and it is always greater than unity and increases as value of n increases, for fixed $\frac{\delta}{R_0}$ Fig. (5) Shows shear stress decreases with the relative length of the constriction.

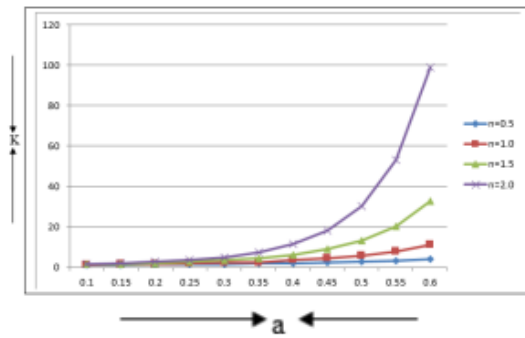


Figure 1. Variation of pressure drop across the length of the stenosis w.r.t. stenosis height at $\varepsilon=0.64$ and $z=2.5$

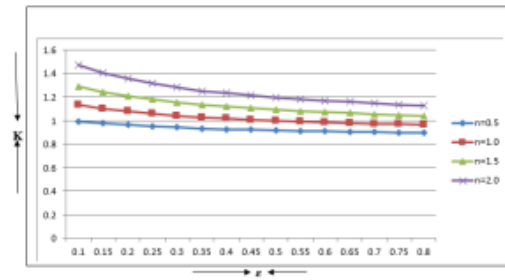


Figure 2. Variation of pressuredrop across the length of the stenosis with the relative length of the constriction

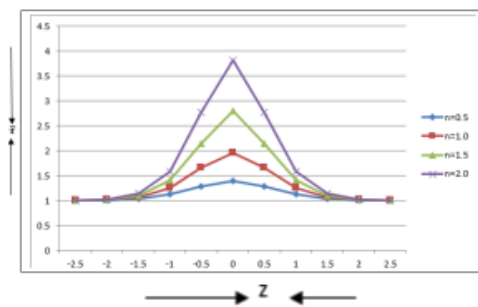


fig. (3) Variation of shear stress along the length of the arterial segment

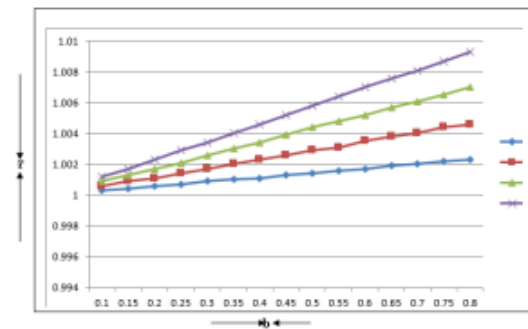


fig.(4) Variation of shear stress with stenosis height

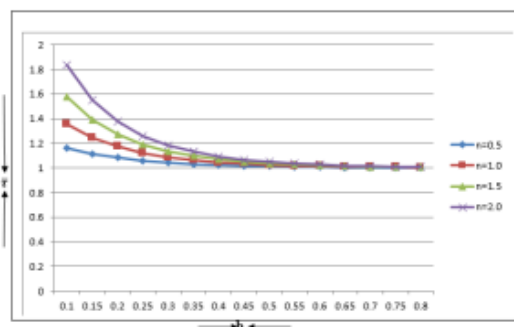


fig.(5) Variation of shear stress with b

This graph between blood pressure drop and hematocrit in major Beta thalassemia (Cooley amania) patient shows that when hematocrit increased then blood pressure drop also increased.

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