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## Ag- Closed Sets In Multiset Topological Spaces

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### Abstract

In general,  $ag$  closed set is a generalization of  $\alpha$ -closed set which was studied in topological spaces. Here in this research article, the notion of  $ag$ - closed sets in multiset topological spaces is introduced and obtained some of its characterizations with examples.

**Key Words:** multiset, sub-mset, Power mset,  $ag$ -closed sets.

**AMS Classification:** 03E70; 54A05;

### 1. Introduction

The mathematical study of a well-defined group of items known as members or elements of a set is known as set theory. Sets are used or referred to in different ways in every branch of mathematics. With the restriction that no components in the set are repeated, they are essential in creating increasingly intricate mathematical structures. However, from a practical standpoint, there are numerous circumstances in which element repetition is crucial. This resulted in the establishment of the theory of Multisets, which Blizard [1] examined for the first time in 1989. A multiset is a group of items in which some items can appear more than once. the multiplicity of an element is the number of times it appears. A through theory of multisets has been developed because of the numerous applications of multisets that have been seen in mathematics and computer science over the years, in addition to philosophy, logic, linguistics, and physics. There are numerous important uses for multisets in various branches of mathematics. Many authors like Yager [11], Miyamoto [7], Hickman [3], Blizard [1] have studied the properties of multisets. An entirely new class of multiset of equivalent pairs was found by Venkateswaram [10]. Singh et.al[9] studied the applications of multi sets,

P. M. Mahalakshmi et. Al[5] studied connectedness in Multi topological spaces and Mahanta et.al[6] studied  $M$ -compactness.

Girish and John [2] thoroughly examined the idea of topological space on a multiset. The concept  $g$ -closed sets in multi set topological spaces was introduced by Binod Tripathy et.al [4] in 2017. Some applications of  $M$ -topology on DNA mutations are also developed by M. El-Sharkasy et.al[8]. Here, the objective of this paperwork is to obtain  $ag$ -closed sets in multi set topological spaces and to obtain few of its characteristics.

### 2. Preliminaries

This section contains some fundamental definitions, symbols that are essential and prelude to the study.

**Definition 2.1:** An mset  $M$  drawn from the set  $X$  is represented by a count function  $M$  or  $C_M : X \rightarrow N$ , where  $N$  represents the set of non-negative integers. We present the  $m$ -set  $M$  drawn from the set  $X = \{x_1, x_2, x_3, \dots, x_n\}$  as  $M = \{m_1/k_1, m_2/k_2, \dots, m_n/k_n\}$ , where  $m_i$  is the occurrences of the element  $x_i$  in the set  $M$ .

**Definition 2.2:** A Sub-mset  $N$  of  $M$  is defined by selecting elements from  $M$  such that the multiplicities of these selected elements in  $N$  do not exceed their multiplicities in  $M$ .

**Definition 2.3:** A sub-mset  $N$  of  $M$  is a multiset over  $S$ , represented by a multiplicity function  $g : S \rightarrow \mathbb{N}$  such that for all  $x \in S$ , the multiplicity  $g(x)$  in  $N$  satisfies  $g(x) \leq f(x)$ , for each  $x \in S$ .

**Definition 2.4:** A sub-mset  $N$  of  $M$  is a Whole sub-mset of  $M$  with each element in  $N$  having full multiplicity as in  $M$ .

**Definition 2.5:** A sub-mset  $N$  of  $M$  is a Partial Whole submset of  $M$  with at least one element in  $N$  having full multiplicity as in  $M$ .

**Definition 2.6:** A Sub-mset  $N$  of  $M$  is a Full submset of  $M$  if  $M$  and  $N$  having the same support set for every  $x$  in  $N$  i.e  $M^* = N^*$ .

**Example 1:** Let us consider the mset  $M = \{9/x, 6/y, 3/z, 5/q\}$  then

- (1) The submset  $N = \{9/x, 3/z, 5/q\}$  is a whole submset and partial whole submset of  $M$ .
- (2) The submset  $N = \{6/x, 4/y, 2/z, 3/q\}$  is a partial whole submset and full submset of  $M$ .
- (3) The submset  $N = \{7/x, 3/z, 2/q\}$  is a partial whole submset of  $M$ .
- (4) The submset  $N = \{2/x, 5/y, 2/z, 3/q\}$  is a full submset of  $M$ .

**Definition 2.7: Power whole mset:** If  $M \in [\square]^m$  be an mset, then the power whole mset of  $M$ , denoted by  $PW(M)$  is the collection of all whole submsets of  $M$ .

**Power full mset:** The collection of all full submsets of an mset is called its power full mset.

**Power set of an mset:** If  $M \in [\square]^m$  be an mset and  $P(M)$  be the power mset, then the power set of  $M$  denoted by  $\square^*(M)$  is the support set of  $P(M)$ .

**Example 2:** If  $M = \{4/x, 3/y\}$  be an mset then

Power whole mset of  $M$  is  $PW(M) = \{\emptyset, M, \{4/x\}, \{6/y\}\}$

Power full mset is  $PF(M) = \{\emptyset, M, \{2/x, 3/y\}, \{3/x, 2/y\}, \{3/x, 3/y\}, \{1/x, 1/y\}\}$

Power set of mset  $M$  is  $P(M) = \{\emptyset, M, \{4/x\}, \{3/y\}, \{4/x\}, \{2/x, 3/y\}, \{3/x, 2/y\}, \{3/x, 3/y\}, \{1/x, 1/y\}, \{4/x, 2/y\}, \{2/x, 1/y\}, \{1/x, 2/y\}\}$

**Definition 2.8:** If  $M_1$  and  $M_2$  are two msets drawn from a set  $X$ , then the cartesian product of  $M_1$  and  $M_2$  is defined as  $M_1 \times M_2 = \{(m/x, n/y)/mn : x \in {}^m M_1, y \in {}^n M_2\}$ .

### Basic Operations:

Consider two msets  $M$  and  $N$  drawn from a set  $X$ .

The following are the operations defined on the msets will be used in this article

- $M = N$  if  $C_M(x) = C_N(x)$  for all  $x \in X$ .
- $M \subseteq N$  if  $C_M(x) \leq C_N(x)$  for all  $x \in X$ .
- $P = M \cup N$  if  $C_P(x) = \max\{C_M(x), C_N(x)\}$  for all  $x \in X$ .
- $P = M \cap N$  if  $C_P(x) = \min\{C_M(x), C_N(x)\}$
- $P = M \oplus N$  if  $C_P(x) = C_M(x) + C_N(x)$  for all  $x \in X$ .
- $P = M \ominus N$  if  $C_P(x) = \max\{C_M(x) - C_N(x), 0\}$  for all  $x \in X$ .

The following are basic operations under collection of msets. Let  $[X]^w$  be an mset space with  $C_z(x)$  as the multiplicities of  $x \in X$  and  $\{M_1, M_2, \dots\}$  be a collection of msets drawn from  $[X]^w$ . Then the following operations are possible under arbitrary collections of msets.

1. The Union  $\cup_{i \in I} M_i = \{C_M(x)/x : C_M(x) = \max\{C_{M_i}(x) : i \in I\}, \text{ for all } x \in X\}$ .
2. The intersection  $\cap_{i \in I} M_i = \{C_M(x)/x : C_M(x) = \min\{C_{M_i}(x) : i \in I\}, \text{ for all } x \in X\}$ .

3. The mset complement  $M^c = Z \ominus M = \{C_M^c(x)/x : C_M^c(x) = C_Z(x) - C_M(x), x \in X\}$ , where  $\oplus$  and  $\ominus$  represent mset addition and mset subtraction respectively.

**Definition 2.9:** Let  $M$  be an mset drawn from a set  $X$ . The support set of  $M$  denoted by  $M^*$  is a subset of  $X$  and  $M^* = \{x \in X : C_M(x) > 0\}$ .

**Remark 1:** An mset  $M$  is said to be an empty set if for all  $x \in X$ ,  $C_M(x) = 0$ .

**Definition 2.10:** Let  $M \in [X]^w$  and  $\tau \subseteq \square^*(M)$ . Then  $\tau$  is called a multiset topology of  $M$  if  $\tau$  satisfies the following properties:

The mset  $M$  and the empty mset  $\emptyset$  are in  $\tau$ .

The mset union of elements of any sub-collection of  $\tau$  belongs to  $\tau$ .

The mset intersection of the elements of any finite sub-collection of  $\tau$  belongs to  $\tau$ .

**Definition 2.11:** Given a submset  $A$  of  $M$ -topological space  $M$  in  $[X]^w$ , the interior of  $A$  is denoted by  $\text{Int}(A)$  and is defined as the mset union of all open msets contained in  $A$ .

i.e.,  $C_{\text{Int}(A)}(x) = \max\{C_G(x) : G \subseteq A\}$ .

**Definition 2.12:** Given a submset  $A$  of an  $M$ -topological space  $M$  in  $[X]^w$ , the closure of  $A$  is defined as the mset intersection of all closed msets containing  $A$  and is denoted by  $\text{Cl}(A)$ , i.e.,  $C_{\text{Cl}(A)}(x) = \min\{C_K(x) : A \subseteq K\}$ .

**Definition 2.13:** Let  $M$  be a multiset drawn from a set  $X$ . A submset  $M_1$  in an  $M$ -topological space  $(M, \tau)$  is said to be a  $g$ -closed mset if,

$$C_{\text{Cl}(M_1)}(x) \leq C_{O_1}(x) \text{ for all } x \in X,$$

whenever,  $C_{M_1}(x) \leq C_{O_1}(x)$ , where  $O_1$  is an open m-set.

### 3. $\alpha g$ - Closed m-sets in Multi set topological spaces

**Definition 3.1:** A subset  $A$  of a multi topological space  $X$  is called  $\alpha$ -closed mset if

$$C_{\text{Cl}(\text{int}(\text{cl}(A)))}(x) \leq C_A$$

The complement of  $\alpha$ -closed set is called  $\alpha$ -open mset.

**Definition 3.2:** A submset  $A$  of multi topological space  $X$  is called  $\alpha g$ -closed mset if  $C_{\alpha \text{Cl}(A)} \subset C_A$ , whenever  $C_A \subset C_U$  and  $C_U$  is open mset in  $X$ .

Shortly,  $C_{\alpha \text{Cl}(A)}(x) = C_A \cup C_{\text{Cl}(\text{int}(\text{cl}(A)))}(x)$

**Example 3:** Let  $X = \{a, b, c, d\}$  and  $M = \{2/a, 3/b, 5/c, 4/d\}$  be an mset.

Then  $\tau = \{\emptyset, M, \{1/a, 2/b\}, \{1/a, 3/b, 3/d\}, \{1/b, 4/c\}, \{1/a, 1/b, 5/c\}, \{1/a, 3/b, 2/c, 3/d\}, \{1/a, 2/b, 2/c, 4/d\}, \{1/b\}, \{2/c\}, \{1/d\}, \{2/c, 2/d\}, \{1/a, 2/b, 4/c\}, \{1/a\}, \{1/a, 3/c\}, \{1/a, 2/b, 2/c\}\}$  is an  $M$ -topology.

Let  $M_1 = \{1/a, 2/b, 1/c, 3/d\}$

$$C_{\alpha \text{Cl}(M_1)}(x) = \{1/a, 2/b, 3/c, 3/d\} \leq C_{O_1}(x) \text{ for all } x \in X.$$

Hence,  $M_1$  is a  $\alpha g$ -closed mset in  $M$ -topological space  $(M, \tau)$ .

**Theorem 1:** The union of two  $\alpha g$ -closed msets is also an  $\alpha g$ -closed mset.

**Proof:** Let  $M$  and  $N$  be  $\alpha g$ -closed msets. Consider an open mset  $M'$  such that

$$C_{M \cup N}(x) \leq C_{M'}(x)$$

Then  $C_M(x) \leq C_{M'}(x)$  and  $C_N(x) \leq C_{M'}(x)$ .

Since  $M$  and  $N$  are  $\alpha g$ -closed mset, we can say that

$$C_{\alpha cl(M)}(x) \leq C_{M'}(x) \text{ and } C_{\alpha cl(N)}(x) \leq C_{M'}(x).$$

$$\begin{aligned} \text{Now, } C_{\alpha cl(M \cup N)}(x) &= \max\{C_{\alpha cl(M)}(x), C_{\alpha cl(N)}(x)\} \\ &\leq \max\{C_M(x), C_N(x)\} \\ &= C_{M'}(x) \end{aligned}$$

Hence  $M \cup N$  is also a  $\alpha g$ -closed mset.

**Remark 2:** Generally, in topological spaces, the intersection of two  $\alpha g$  closed sets need not be  $\alpha g$ -closed, but in the case of multi topological spaces, the intersection of two  $\alpha g$ -closed msets is again an  $\alpha g$ -closed mset.

**Example 4:** Let  $A$  and  $B$  be  $\alpha g$ -closed msets. Consider an open mset  $M'$  such that

$$C_{A \cap B}(x) \leq C_{M'}(x)$$

Then  $C_A(x) \leq C_{M'}(x)$  and  $C_B(x) \leq C_{M'}(x)$ .

Since  $A$  and  $B$  are  $\alpha g$ -closed mset, we can say that

$$C_{\alpha cl(A)}(x) \leq C_{M'}(x) \text{ and } C_{\alpha cl(B)}(x) \leq C_{M'}(x).$$

$$\text{Now, } C_{\alpha cl(A \cap B)}(x) = \min\{C_{\alpha cl(A)}(x), C_{\alpha cl(B)}(x)\} \leq C_{M'}(x)$$

Hence  $M \cap N$  is also a  $\alpha g$ -closed mset.

**Example 5:** In example 3.3,  $M_1 = \{1/a, 2/b, 1/c, 3/d\}$  and  $M_2 = \{2/a, 2/d\}$  are  $\alpha g$ -closed msets. Their intersection  $\{1/a, 2/d\}$  is again an  $\alpha g$ -closed mset.

**Remark 3:** Every  $m$ -Closed mset is  $\alpha g$ -Closed mset but the converse may not be true.

**Example 6:** Let  $X = \{a, b, c, d\}$  and  $M = \{3/a, 2/b, 4/c, 6/d\}$  be an mset in  $[\square]^6$ .

Then  $\tau = \{\emptyset, M, \{2/a\}\}$  is an  $M$ -topology. Here the subset  $\{1/a, 3/b\}$  is  $\alpha g$ -Closed mset but not closed mset.

**Remark 4:** Every  $g$ -closed mset is  $\alpha g$ -Closed mset but the converse may not be true.

**Example 7:** In example 3.8, the subset  $\{2/b, 3/c\}$  is  $\alpha g$ -Closed mset but not  $g$  closed mset.

**Theorem 2:** Let  $M$  be a topological space in  $[X]^w$ . If  $N$  is any  $\alpha g$ -closed mset in  $M$  and  $C_N(x) \leq C_Y(x) \leq C_{\alpha cl(N)}(x)$ , then  $Y$  is  $\alpha g$ -closed.

**Proof:** Let  $O$  be any open mset such that,  $C_Y(x) \leq C_O(x)$

From the given condition,  $C_N(x) \leq C_O(x)$

But  $N$  is  $\alpha g$ -closed and so we have,  $C_{\alpha cl(N)}(x) \leq C_O(x)$

Since,  $C_Y(x) \leq C_{\alpha cl(N)}(x) \leq C_O(x)$

Hence  $Y$  is  $\alpha g$ -closed.

**Theorem 3:** An mset  $M_2$  is  $\alpha g$ -open if and only if  $C_F(x) \leq C_{\alpha int(M_2)}(x)$  for all  $x \in X$ , whenever  $F$  is a closed mset and  $C_F(x) \leq C_{M_2}(x)$  for all  $x \in X$ .

**Proof :** Let  $M_2$  be a  $\alpha g$ -open mset. Then  $M_2^c$  is  $\alpha g$ -closed.

So,  $C_{\alpha cl(M_2^c)}(x) \leq C_O(x)$  for all  $x \in X$ ,

whenever  $C_{(M_2^c)}(x) \leq C_O(x)$  for all  $x \in X$ , where  $O$  is open.

But,  $C_{\alpha cl(M_2^c)}(x) = C_{(\alpha int M_2)^c}(x)$  for all  $x \in X$ .

Hence (4) becomes,

$$C_{(\alpha int M_2)^c}(x) \leq C_O(x) \text{ for all } x \in X, \text{ whenever } C_{(M_2^c)}(x) \leq C_O(x) \text{ for all } x \in X.$$

By De-Morgan's law,

$$C_{(O^c)}(x) \leq C_{(\alpha int M_2)^c}(x), \text{ whenever } O^c = F \text{ (Since } O \text{ is open) is closed and}$$

$$C_{(O^c)}(x) \leq C_{M_2}(x)$$

Conversely, let there be a closed mset  $F$  such that the given condition holds. Then, by De-Morgan's law we have,

$$C_{(\alpha int M_2)^c}(x) \leq C_{(F^c)}(x), \text{ whenever } C_{(M_2^c)}(x) \leq C_{(F^c)}(x) \text{ for all } x \in X.$$

We have,  $C_{\alpha cl(M_2^c)}(x) \leq C_{(F^c)}(x)$  whenever  $C_{\alpha cl(M_2^c)}(x) \leq C_{(F^c)}(x) \forall x \in X$  ( $F^c$  is open).

Hence  $M_2^c$  is  $\alpha g$ -closed, so  $M_2$  is a  $\alpha g$ -open mset.

**Theorem 4:** Let  $C_{\alpha int M} \leq C_N \leq C_M$  and if  $M$  is  $\alpha g$ -open, then  $N$  is  $\alpha g$ -open.

**Proof:** By hypothesis we have,  $C_{\alpha int M} \leq C_N \leq C_M$ .

By De-Morgan's law,  $C_{M^c} \leq C_{N^c} \leq C_{(\alpha int M)^c} = C_{(\alpha cl(M)^c)}$

Since  $M$  is  $\alpha g$ -open,  $M^c$  is  $\alpha g$ -closed. Hence it follows from theorem 3.12 that  $N^c$  is a  $\alpha g$ -closed mset i.e.  $N$  is  $\alpha g$ -open mset.

**Theorem 5:** If  $M$  is open submset and  $\alpha g$ -closed mset, the  $A$  is  $\alpha$ -clopen mset.

**Proof:** Let  $M$  be an open and  $\alpha g$ -closed m set. As every open mset is  $\alpha$ -open mset and  $M$  is contained in  $M$ , we have  $\alpha cl(M) \subset M$ . Further as  $M \subset \alpha cl(M)$ ,  $M = \alpha cl(M)$ . Hence  $M$  is  $\alpha$ -clopen.

**Theorem 6:** If  $M_2$  is a  $\alpha g$ -closed submset of  $M$  then  $M_2 \subset M_1 \subset \alpha cl(M_2)$  then  $M_1$  is  $\alpha g$ -closed set in  $M$ .

**Proof :** If  $M_2$  is  $\alpha g$ -closed submset of  $M$  such that  $M_2 \subset M_1 \subset \alpha cl(M_2)$ .

Let  $N$  be a  $\alpha g$ -open set of  $M$  such that  $M_1 \subset N$

Then  $M_2 \subset N$ . Since  $M_2$  is a  $\alpha g$ -closed.

we have,  $\alpha cl(M_2) \subset N$

Now,  $\alpha cl(M_1) \subset \alpha cl(\alpha cl(M_2)) = \alpha cl(M_2) \subset N$

Therefore  $M_1$  is  $\alpha g$ -closed set in  $M$ .

**Theorem 7:** Let  $M_1$  be a  $\alpha g$ -closed in Multi topological spaces  $M$ . Then  $M_1$  is  $\alpha$ -closed iff  $\alpha cl(M_1) \setminus M_1$  is  $\alpha g$ -open.

**Proof:** Suppose  $M_1$  is a  $\alpha$ -closed in  $X$ .

Then  $\alpha cl(M_1) = M_1$  and so  $\alpha cl(M_1) \setminus M_1 = \emptyset$ , which is  $\alpha$ -open in  $X$ .

Conversely, Suppose,  $\alpha cl(M_1) \setminus M_1$  is a  $\alpha$ -open set in  $X$ .

Since  $M_1$  is  $\alpha g$ -closed,  $\alpha cl(M_1) \setminus M_1$  does not contain any nonempty  $\alpha$ -open in  $X$ .

Then,  $\alpha cl(M_1) \setminus M_1 = \emptyset$ , hence  $M_1$  is  $\alpha$ -closed set in  $X$ .

## CONCLUSION

From the perspective of multiset topological space, we have examined few characteristics of the concepts of  $\alpha$ -generalized closed sets. The future scope of the research work is to analyse, obtain some more characteristics of this kind of closed mset by comparing with general topological spaces in all aspects like the connectedness and compactness.

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