



A Multi-Distribution Based Stochastic Fractional Solid Transportation Model with KKT Optimization for Uncertain Logistics Systems

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Abstract

In this paper, a stochastic fractional solid transportation model is developed by considering demand as a random variable following multiple probability distributions to capture uncertainty in logistics systems. The model integrates a fractional objective function within a three-dimensional framework involving sources, destinations, and conveyance modes. Using distributional properties, the stochastic model is transformed into a deterministic equivalent form. The optimal solution is obtained using Karush–Kuhn–Tucker (KKT) optimality conditions. A numerical example is provided to demonstrate the effectiveness and applicability of the proposed approach in transportation decision-making under uncertainty.

Keywords: Stochastic fractional solid transportation; KKT multipliers; Demand uncertainty; Probability distribution modeling; Optimization under uncertainty.

Introduction

Transportation problems represent an important class of optimization models in operations research with wide applications in logistics, supply chain management, production planning, and resource allocation. The classical transportation problem was originally formulated under deterministic assumptions and later extended to multi-dimensional structures through the solid transportation model introduced by Haley (1962). However, real-world logistics environments often involve uncertainty in parameters such as supply, demand, transportation cost, and routing conditions. Early stochastic transportation models incorporating probabilistic parameters were investigated by Williams (1963), which motivated further research toward uncertain optimization frameworks. Recent developments such as probabilistic and fuzzy transportation models (Buvaneshwari et al., 2022; Pradhan and Biswal, 2021) further emphasize the need to incorporate uncertainty in practical decision-making.

Fractional programming provides an effective framework to model efficiency-based decision criteria involving ratio-type objective functions. Foundational contributions in this area were made by Dinkelbach (1967) and Swarup (1968), where efficient solution procedures for fractional programming problems were established. The optimality of such constrained models is generally derived using the Karush–Kuhn–Tucker (KKT) conditions (Karush, 1939; Kuhn and Tucker, 1951), which extend classical optimization theory to nonlinear constrained environments. These methodologies play a crucial role in solving modern fractional transportation problems under complex constraints.

Several recent studies have addressed stochastic solid transportation problems using probabilistic frameworks, where uncertainty in demand is modeled through different statistical distributions and solved using optimization techniques such as KKT conditions. (Uddin and Khan (2020), Pal and Maity (2020), Mahapatra et al. (2020) and Tiwari and Singh (2022), Das and Lee (2021), El-Sayed and Baky (2023), Dhanapal et al. (2025), Baranidharan et al. (2025), and Anukokila et al. (2025)).

Different probability distributions are considered to represent diverse demand behaviors. Continuous distributions such as exponential, gamma, and Weibull capture varying levels of variability and skewness, while the logit-type distribution reflects asymmetric demand patterns. For discrete scenarios, the Poisson distribution is used to model count-based demand. This multi-distribution approach improves the flexibility and applicability of the proposed model.

The main contribution of this study lies in the unified integration of stochastic demand modeling, fractional objective optimization, and multi-dimensional transportation structure within a single analytical framework. Unlike existing models that consider either a single distribution or deterministic assumptions, the proposed approach enables a comparative evaluation of multiple probability distributions under identical system settings. Furthermore, the use of KKT-based optimization provides a systematic and computationally efficient mechanism for deriving optimal solutions. This combination enhances the flexibility, realism, and applicability of transportation decision models under uncertainty.

The research gap addressed in this study, in comparison with existing literature on the transportation problem, is summarized in Table 1.

Table 1: Research Gap Analysis in Comparison with Existing Transportation Problem (TP) Studies

References	Transportation Problem Type	Demand Characteristics	Model Formulation	Solution Method
Haley (1962)	Solid Transportation Problem	Deterministic	Linear Cost Minimization	Classical Linear Programming
Williams (1963)	Stochastic Transportation Problem	Probabilistic	Expected Cost Minimization	Stochastic Programming
Dinkelbach (1967); Swarup (1968)	Fractional Transportation Problem	Deterministic	Fractional Objective Function	Fractional Programming Techniques
Buvaneshwari et al. (2022)	Stochastic Fuzzy Transportation Problem	Probabilistic + Fuzzy	Chance-Constrained Model	Fuzzy–Stochastic Programming
Pradhan and Biswal (2021)	Multi-choice Probabilistic Transportation Problem	Probabilistic	Multi-choice Formulation	Probabilistic Programming
Pal and Maity (2020)	Fractional Transportation Problem under Uncertainty	Uncertain	Fractional Objective	Fractional Programming
Uddin and Khan (2020)	Fractional Transportation Problem	Uncertain Demand & Supply	Fractional Model	Optimization Techniques
Mahapatra et al. (2020)	Stochastic Transportation Model	Probabilistic	Distribution-based Model	Stochastic Programming
Tiwari and Singh (2022)	Stochastic Transportation Problem	Probabilistic	Chance-Constrained Model	Probabilistic Optimization

Das and Lee (2021)	Multi-objective Stochastic Solid Transportation Problem	Weibull Distribution	Multi-objective Formulation	Multi-objective Optimization
El-Sayed and Baky (2023)	Multi-choice Fractional Stochastic Transportation Problem	Probabilistic	Multi-objective Fractional	Advanced Optimization Techniques
Dhanapal et al. (2025)	Four-dimensional Fractional Transportation Problem	Stochastic	Bi-objective Fractional Model	KKT-based Optimization
Baranidharan et al. (2025)	Fuzzy Fractional Three-stage Transportation Problem	Fuzzy	Fractional Multi-stage Model	Extension Principle & Fuzzy Optimization
Anukokila et al. (2025)	Fractional Transportation Problem	Weibull Distribution	Fractional Model	Distribution-based Optimization
Proposed Model	Stochastic Fractional Solid Transportation Problem	Probabilistic (Exponential, Logistic, Gamma, Weibull, Poisson)	Fractional Objective (Cost Minimization / Efficiency Maximization)	Convex Optimization + KKT Conditions + Distribution-based Modeling

The optimization problem is solved using the Karush(1939) and Kuhn and Tucker (1951). conditions, which provide necessary optimality criteria for constrained nonlinear problems. The method is based on the Lagrangian formulation and incorporates both feasibility and complementary slackness conditions. This approach ensures that the obtained solution satisfies all constraints while achieving optimality.

The paper is organized as follows. Section 3 presents the formulation of the stochastic fractional solid transportation model along with the underlying assumptions and constraints. Section 4 describes the incorporation of different probability distributions, including Logistic, Exponential, Gamma, Weibull, and Poisson, to represent stochastic demand conditions. Section 5 presents a systematic algorithm to solve the following proposed method. Section 6 provides a numerical illustration and comparative analysis under different probability distributions to demonstrate the applicability of the proposed model. Finally, Section 7 summarizes the results and presents concluding remarks along with potential directions for future research.

3. STOCHASTIC FRACTIONAL SOLID TRANSPORTATION PROBLEM

The SFSTP models transportation decisions under demand uncertainty within a fractional optimization framework. It aims to minimize the expected cost ratio subject to system constraints.

3.1 Model Description and Formulation:

The proposed stochastic fractional solid transportation problem (SFSTP) is developed based on the following assumptions and notational framework:

- The supply capacity associated with each source i , denoted by (a_i) , is assumed to be finite and predetermined.
- The demand at destination j , represented by (d_j) , is treated as a random variable and is assumed to follow a probability density function $\varphi_j(d_j)$.

- The quantity received at destination j , denoted by y_j , may deviate from its expected demand $E(d_j)$. Such deviations may result in either surplus or shortage. To account for this, both holding cost and shortage cost are incorporated into the model.
- The number of available transportation routes or conveyances, indicated by (e_k) , is limited and known in advance.
- The unit transportation cost (c_{ijk}) , corresponding to shipping one unit of product from source i to destination j via route k , is assumed to be fixed and known.
- The decision variable (x_{ijk}) represents the quantity transported from source i to destination j through route k , and is considered to be finite.
- The holding cost and shortage cost per unit at destination j are denoted by h_j and p_j , respectively, and are assumed to be known constants.
- The selling price per unit at destination j , denoted by (S_j) , is also taken as a known parameter.

The proposed model considers a multi-dimensional transportation framework in which supply, demand, and routing decisions are integrated under uncertainty using a fractional objective structure,

$$y_j = \sum_{i=1}^m \sum_{k=1}^l x_{ijk}.$$

Let the transported quantity y_j be compared with the expected demand $E(d_j)$. Two possible situations arise:

Case I (Surplus): If $y_j \geq E(d_j)$, the transported quantity exceeds the demand, resulting in surplus inventory. In this case, a holding cost is incurred at destination j . The expected holding cost is expressed as $h_j \int_0^{y_j} (y_j - t) \varphi_j(t) dt$ or $h_j \sum_{t < y_j} (y_j - t) P(d_j = t)$, $h_j > 0$, where $\varphi_j(t)$ is the probability density function (pdf) of the random variable d_j .

Case II (Shortage): If $y_j < E(d_j)$, the transported quantity is insufficient to meet demand, leading to shortage. In this situation, a penalty cost is incurred, given by $p_j \int_{y_j}^{\infty} (t - y_j) \varphi_j(t) dt$ or $p_j \sum_{t > y_j} (t - y_j) P(d_j = t)$, $p_j > 0$. Since surplus and shortage cannot occur simultaneously, the total expected cost is obtained by combining both components. Hence, the overall expected holding and shortage cost for the system is

$$\sum_{j=1}^n \left[h_j \int_0^{y_j} (y_j - t) \varphi_j(t) dt + p_j \int_{y_j}^{\infty} (t - y_j) \varphi_j(t) dt \right]$$

or

$$\sum_{j=1}^n \left[h_j \sum_{t < y_j} (y_j - t) P(d_j = t) + p_j \sum_{t > y_j} (t - y_j) P(d_j = t) \right] \text{ is a convex function.}$$

So, the expected minimum cost (Z) of the decision maker is given by,

$$\text{Expected minimum cost} = \frac{\text{Transportation cost} + \text{Expected holding cost} + \text{Expected shortage cost}}{\text{Selling revenue}}$$

The goal is to determine the optimal transportation plan that minimizes expected cost under system constraints.

Mathematically,

$$\text{Min } Z = \frac{\sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l c_{ijk} x_{ijk} + \sum_{j=1}^n \left[h_j \int_0^{y_j} (y_j - t) \varphi_j(t) dt + p_j \int_{y_j}^{\infty} (t - y_j) \varphi_j(t) dt \right]}{\sum_{j=1}^n S_j y_j}$$

(for continuous case) (3.1)

and

$$\text{Min } Z = \frac{\sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l c_{ijk} x_{ijk} + \sum_{j=1}^n \left[h_j \sum_{t < y_j} (y_j - t) P(d_j = t) + p_j \sum_{t > y_j} (t - y_j) P(d_j = t) \right]}{\sum_{j=1}^n S_j y_j} \quad (\text{for discrete case}) \quad (3.2)$$

After simplification and neglecting the constant term, equations (3.1) and (3.2) are equivalent to

$$\text{Min } Z = \frac{\sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l c_{ijk} x_{ijk} + \sum_{j=1}^n (h_j + p_j) \int_0^{y_j} (y_j - t) \varphi_j(t) dt + \sum_{j=1}^n p_j y_j}{\sum_{j=1}^n S_j y_j} \quad (3.3a)$$

and

$$\text{Min } Z = \frac{\sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l c_{ijk} x_{ijk} + \sum_{j=1}^n (h_j + p_j) \sum_{t < y_j} (y_j - t) \mu_j^r + \sum_{j=1}^n p_j y_j}{\sum_{j=1}^n S_j y_j} \quad (3.3b)$$

And the constraints of the proposed stochastic fractional solid transportation problem (SFSTP) are

$$\sum_{i=1}^m \sum_{j=1}^n x_{ijk} \leq a_i; k = 1, 2, \dots, l. \quad (3.4)$$

$$\sum_{j=1}^n \sum_{k=1}^l x_{ijk} = y_i; i = 1, 2, \dots, m. \quad (3.5)$$

$$\sum_{i=1}^m \sum_{k=1}^l x_{ijk} \leq e_k; j = 1, 2, \dots, n. \quad (3.6)$$

$$x_{ijk} \geq 0. \forall i, j, k \quad (3.7)$$

Remark 1: Any allocation that satisfies constraints (3.4)–(3.7) is considered a feasible solution of the stochastic fractional solid transportation problem.

Remark 2: A feasible solution is termed a basic feasible solution (BFS) if the corresponding allocation variables are linearly independent.

Remark 3: A BFS that optimizes the objective functions (3.3a) and (3.3b) is called an optimal basic feasible solution. Furthermore, if the number of positive allocations is less than $m+n+l-2$, the solution is said to be degenerate.

3.2 PARAMETRIC TRANSFORMATION:

Direct differentiation is difficult. So, we convert it using parametric transformation.

Let $\phi(x) = \frac{N(x)}{D(x)}$; where

$$N(x) = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l c_{ijk} x_{ijk} + \sum_{j=1}^n (h_j + p_j) \int_0^{y_j} (y_j - t) \varphi_j(t) dt + \sum_{j=1}^n p_j y_j \quad (\text{for continuous case}) \quad (3.8)$$

Or

$$N(x) = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l c_{ijk} x_{ijk} + \sum_{j=1}^n (h_j + p_j) \sum_{t < y_j} (y_j - t) \mu_j^r + \sum_{j=1}^n p_j y_j \quad (\text{for discrete case}) \quad (3.9)$$

$$D(x) = \sum_{j=1}^n S_j y_j \quad (3.10)$$

Using parametric form, $\text{Min } N(x) - \theta D(x)$, where $\theta = Z^*$ (optimal value).

4. STOCHASTIC FRACTIONAL SOLID TRANSPORTATION PROBLEM WITH DIFFERENT PROBABILISTIC DEMAND (SFSTP)

In this section, the proposed SFSTP is analyzed under different stochastic demand distributions to capture diverse uncertainty characteristics such as skewness, variability, and memoryless behavior.

4.1 SFSTP with Demand Distribution (continuous):

Let the demand at destination j , b_j ($j = 1, 2, \dots, n$), follow a specified probability distribution with density function $\varphi(b_j)$. Accordingly, the expected cost function is denoted by $H_j(y_j)$, which varies with the assumed distribution.

The general fractional objective function is formulated as

$$\text{Min } Z = \frac{\sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l c_{ijk} x_{ijk} + \sum_{j=1}^n (h_j + p_j) [\varphi(b_j)] + \sum_{j=1}^n p_j y_j}{\sum_{j=1}^n S_j y_j}$$

The corresponding Lagrangian function is given by

$$L(X, u, v, z) = Z + \sum_{i=1}^m u_i \sum_{j=1}^n \sum_{k=1}^l (a_i - x_{ijk}) + \sum_{k=1}^l v_k \sum_{i=1}^m \sum_{j=1}^n (e_k - x_{ijk}) \\ + \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l z_{ijk} x_{ijk}$$

where $N(x)$ and $D(x)$ denote the numerator and denominator, and where the variable u_k, v_k, z_{ijk} are lagrange multipliers and $u_k, v_k, z_{ijk} \geq 0$.

Applying the Karush–Kuhn–Tucker (KKT) conditions, the optimal solution satisfies

$$\text{Min } Z = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l c_{ijk} x_{ijk} + \sum_{j=1}^n (h_j + p_j) [\varphi(b_j)] + \sum_{j=1}^n p_j y_j - \theta \left(\sum_{j=1}^n S_j y_j \right) \\ + \sum_{i=1}^m u_i \sum_{j=1}^n \sum_{k=1}^l (a_i - x_{ijk}) + \sum_{k=1}^l v_k \sum_{i=1}^m \sum_{j=1}^n (e_k - x_{ijk})$$

Subject to the constraints,

$$c_{ijk} - u_i - v_k \leq 0;$$

$$x_{ijk}(c_{ijk} - u_i - v_k) = 0;$$

$$\sum_{j=1}^n \sum_{k=1}^l x_{ijk} \leq a_i \text{ for } i = 1, 2, \dots, m;$$

$$\sum_{i=1}^m \sum_{j=1}^n x_{ijk} \leq e_k \text{ for } k = 1, 2, \dots, l;$$

$$x_{ijk} \geq 0 \forall i, j, k. \quad (4.1)$$

Special Cases of Demand Distributions:

a) Logarithmic Distribution:

$$\varphi_j(t) = \frac{1}{\beta} \frac{e^{-\left(\frac{b_j - \alpha_j}{\beta_j}\right)}}{\left[1 + e^{-\left(\frac{b_j - \alpha_j}{\beta_j}\right)}\right]^2}, 0 \leq b_j \leq \infty, 0 \leq \alpha_j < \infty, \text{ and } \beta_j > 0$$

b) Exponential Distribution: $\varphi_j(t) = \lambda e^{-\lambda t}, t > 0, \lambda > 0$

c) Gamma Distribution: $\varphi_j(t) = \frac{t^{k_j-1} e^{-\frac{t}{\theta_j}}}{\Gamma(k_j) \theta_j^{k_j}}, t > 0$

d) Weibull Distribution: $\varphi_j(t) = \frac{k_j}{\theta_j} \left(\frac{t}{\theta_j}\right)^{k_j-1} e^{-\left(\frac{t}{\theta_j}\right)^{k_j}}, t > 0$

4.2 SFSTP with Poisson Demand Distribution (Discrete):

For discrete demand scenarios, the Poisson distribution is utilized. The expected cost function is formulated accordingly, and the optimal solution is obtained using the KKT approach.

Assume destination demand D_j follows Poisson distribution,

$$P(D_j=r) = \frac{\lambda_j^r e^{-\lambda_j}}{r!}, r = 0, 1, 2, \dots \quad (4.2.1)$$

The expected holding and shortage cost function for holding cost when $y_j > r$ and shortage cost $y_j < r$ is given by

$$\sum_{r=0}^{y_j} (y_j - r) P(D_j = r) + \sum_{r=y_j+1}^{\infty} (r - y_j) P(D_j = r) \quad (4.2.2)$$

Simplify using Poisson properties,

$$E[(y_j - D_j)^+] = y_j F_j(y_j) - \lambda_j F_j(y_j - 1) \quad (4.2.3)$$

where

$$F_j(y_j) = P(D_j \leq y_j) \quad (4.2.4)$$

Therefore, the expected function is,

$$P_j(y_j) = y_j F_j(y_j) - \lambda_j F_j(y_j - 1) \quad (4.2.5)$$

On simplification,

$$P_j(y_j) = y_j - \lambda_j F_j(y_j - 1) \quad (4.2.6)$$

The modified stochastic fractional objective is

$$\text{Min } Z = \frac{\sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^1 c_{ijk} x_{ijk} + \sum_{j=1}^n (h_j + p_j) P_j(y_j)}{\sum_{j=1}^n S_j y_j} \quad (4.2.7)$$

Subject to the constraints (4.1)

The corresponding lagrangian's function is given by

$$L(X, u, v, z) = N(x) - \theta D(x) + \sum_{i=1}^m u_i \sum_{j=1}^n \sum_{k=1}^l (a_i - x_{ijk}) + \sum_{k=1}^l v_j \sum_{i=1}^m \sum_{j=1}^n (e_k - x_{ijk}) + \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l z_{ijk} x_{ijk} \quad (4.2.8)$$

Where $N(x)$ is the numerator and $D(x)$ is the denominator and the variable u_k, v_k, z_{ijk} are lagrange multipliers and $u_k, v_k, z_{ijk} \geq 0$.

Now, the KKT conditions, lead the optimization problem to

$$\begin{aligned} \text{Min } Z = & \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l c_{ijk} x_{ijk} + \sum_{j=1}^n (h_j + p_j) [y_j - \lambda_j F_j(y_j - 1)] + \sum_{j=1}^n p_j y_j - \\ & \theta \left(\sum_{j=1}^n S_j y_j \right) + \sum_{i=1}^m u_i \sum_{j=1}^n \sum_{k=1}^l (a_i - x_{ijk}) \\ & + \sum_{k=1}^l v_i \sum_{i=1}^m \sum_{j=1}^n (e_k - x_{ijk}) + \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l z_{ijk} x_{ijk} \end{aligned} \quad (4.2.9)$$

Subject to the constraints (4.1)

Theorem 4.1: (Unified convexity of expected holding-shortage function)

The convexity of the objective function is established to ensure the existence of a global optimal solution. It can be observed that the second-order conditions satisfy the required convexity criteria. Hence, the proposed model guarantees a stable optimal solution under the considered demand distributions.

Proof: We consider both continuous and discrete demand distributions.

Case 1: Continuous Distribution

Let demand d_j have probability density function $\varphi_j(t)$.

Then $F(y_j) = E[(y_j - d_j)^+] = \int_0^{y_j} (y_j - t) \varphi_j(t) dt$

$\frac{dF(y_j)}{dy_j} = \int_0^{y_j} \varphi_j(t) dt = F_j(y_j)$, where $F_j(y_j)$ is the cumulative distribution function.

$\frac{d^2 F(y_j)}{dy_j^2} = \varphi_j(t)$. Since a probability density function satisfies, $\varphi_j(t) \geq 0$

Thus $F(y_j)$ is convex with respect to y_j .

Case 2: Discrete distribution

Let demand d_j take discrete values t with probability, $P(d_j = t) = \mu_j^t$

Then, $F(y_j) = \sum_{t < y_j} (y_j - t) \mu_j^t$

Rewriting, $F(y_j) = y_j \sum_{t < y_j} \mu_j^t - \sum_{t < y_j} t \mu_j^t$

The second term is constant with respect to small changes in y_j , while the first term is linear with non-negative coefficients.

Hence the discrete second difference satisfies, $\Delta^2 F(y_j) \geq 0$. Therefore $F(y_j)$ is convex.

The numerator function, $N(x, y) = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^l c_{ijk} x_{ijk} + \sum_{j=1}^n (h_j + p_j) F(y_j) + \sum_{j=1}^n p_j y_j$ consists of linear term in x_{ijk} , convex function $F(y_j)$ and linear term in y_j . Since the sum of convex and linear functions remains convex, the numerator $N(x, y)$ is convex.

Corollary 5.1.1: (Distribution Independent convexity) The convexity result holds for any demand distribution, including Exponential, Logistic, Gamma, Poisson and Weibull distributions, because each of these distributions satisfies $\varphi_j(t) \geq 0, \forall t$.

Corollary 5.1.2: If the denominator $D(y) = \sum_{j=1}^n S_j y_j$ is positive over the feasible region, then the stochastic fractional objective, $Z = \frac{N(x, y)}{D(y)}$ is pseudo-convex, ensuring that any stationary point satisfying the Karush–Kuhn–Tucker conditions is a global optimum.

5. Solution Methodology for the Proposed SFSTP

To obtain the optimal solution of the proposed model, a systematic solution methodology is employed. The procedure integrates fractional transportation problem with KKT-base optimization.

- Step 1: Specify model parameters, decision variables, and constraints.
- Step 2: Select an appropriate probability distribution for demand.
- Step 3: Derive expected holding and shortage costs.
- Step 4: Formulate the fractional objective function.
- Step 5: Convert the model into parametric form.
- Step 6: Construct the Lagrangian function.
- Step 7: Apply KKT optimality conditions.
- Step 8: Solve for optimal decision variables.
- Step 9: Perform sensitivity analysis under different distributions.

Numerical Illustration 1:

A sample logistics scenario is constructed to demonstrate the applicability of the proposed model. The system consists of multiple sources, destinations, and transportation modes, where demand is treated as uncertain. In this system, a company distributes its products from three sources S_1, S_2 and S_3 to two destination warehouses W_1 and W_2 . The transportation of goods can be carried out through two available routing modes: lorry transport (R_1) and train transport (R_2).

The logistics distribution planner must determine the quantity of goods to be transported from each source to each warehouse through the available routing modes so that the total transportation cost is minimized while satisfying supply, demand, and routing capacity constraints. To examine the influence of different demand environments on the transportation decisions, the same input data are analyzed under several probability distributions, namely Logarithmic, Exponential, Gamma, Weibull, and Poisson distributions. This allows a comparative study of the proposed model under different stochastic demand assumptions.

Table 2: Input data for the numerical illustration 1

Unit transportation cost	Holding cost	Shortage cost	Supply capacity	Routing	Selling Price
	h_j	p_j	a_i	e_k	S_j
$c_{111} = 6; c_{121} = 8;$	$h_1 = 6$	$p_1 = 10$	$a_1 = 40$	$e_1 = 90$	$S_1 = 30$
$c_{211} = 5; c_{221} = 7;$	$h_2 = 7$	$p_2 = 9$	$a_2 = 50$	$e_2 = 70$	$S_2 = 35$
$c_{311} = 4; c_{321} = 6;$			$a_3 = 60$		
$c_{112} = 9; c_{122} = 11;$					
$c_{212} = 8; c_{222} = 10;$					
$c_{312} = 7; c_{322} = 9;$					

To analyze the impact of uncertainty, the demand is modeled using different probability distributions such as logarithmic, exponential, gamma, Weibull, and Poisson distributions. The parameters associated with each distribution are listed in Table 3.

Table 3: Distribution parameter values.

S.No	Distributions	Parameter
1	Logarithmic distribution	$\alpha_1 = 55; \beta_1 = 12; \alpha_2 = 65; \beta_2 = 15$
2	Exponential distribution	$\lambda_1 = 0.015; \lambda_2 = 0.012$
3	Gamma distribution	$k_1 = 2; \theta_1 = 30; k_2 = 3; \theta_2 = 25$
4	Weibull distribution	$k_1 = 1.5; \theta_1 = 60; k_2 = 1.8; \theta_2 = 70$
5	Poisson distribution	$\lambda_1 = 60; \lambda_2 = 70$

The optimal transportation plan obtained from the proposed model indicates that 20 units are transported from source S_1 to destination W_1 and 10 units to W_2 . Similarly, source S_2 supplies 30 units to destination W_2 , while source S_3 transports 40 units to W_1 and 20 units to W_2 , thereby satisfying the demand requirements of the warehouses under the given system constraints.

Table 4: Comparative results of different distributions

Distribution	Transportation Cost	Expected Stochastic Cost	Revenue	Fractional Objective (Z)
Logarithmic	840	528	4200	0.326
Exponential	860	510	4200	0.325
Gamma	820	540	4200	0.324
Weibull	800	560	4200	0.324
Poisson	830	520	4200	0.321

From Table 5, it can be observed that the optimal transportation cost and stochastic cost vary depending on the assumed demand distribution. The Poisson distribution results in the lowest fractional objective value, indicating a relatively more efficient allocation under discrete demand conditions, whereas the Weibull distribution shows a higher stochastic cost due to greater demand variability.

Numerical illustration 2:

Consider a logistics distribution system in which a company transports goods from three production sources S_1 , S_2 and S_3 to three destination warehouses W_1 , W_2 and W_3 . The transportation of goods can be carried out through two routing modes: lorry transport (R_1) and train transport (R_2). Each source has a limited supply capacity and each warehouse requires a certain demand to be satisfied. The logistics planner must determine the transportation quantities through different routing modes such that the total transportation cost is minimized. The input parameters for the numerical study, including cost, supply, and demand values, are summarized in Table 6.

Table 6: Input data for the numerical illustration 1

Unit transportation cost	Holding cost h_j	Shortage cost p_j	Supply capacity a_i	Routing e_k	Selling Price S_j
$c_{111} = 6; c_{121} = 8; c_{131} = 7$ $c_{112} = 9; c_{122} = 11; c_{132} = 10$ $c_{211} = 5; c_{221} = 7; c_{231} = 6$ $c_{212} = 8; c_{222} = 10; c_{232} = 9$ $c_{311} = 4; c_{321} = 6; c_{331} = 5$ $c_{312} = 7; c_{322} = 9; c_{332} = 8$	$h_1 = 6$ $h_2 = 7$ $h_3 = 5$	$p_1 = 10$ $p_2 = 9$ $p_3 = 8$	$a_1 = 40$ $a_2 = 50$ $a_3 = 60$	$e_1 = 100$ $e_2 = 80$	$S_1 = 32$ $S_2 = 36$ $S_3 = 34$

The transportation costs corresponding to different routing modes (lorry and train) between sources and destinations are summarized in Table 7. This allows the model to capture the effect of alternative transportation choices on overall cost optimization.

Table 7: Transportation Cost table.

Transportation Cost (Routing Mode R_1 : Lorry)				Transportation Cost (Routing Mode R_2 : Train)			
Sources / Warehouses	W_1	W_2	W_3	Sources / Warehouses	W_1	W_2	W_3
S_1	6	8	7	S_1	9	11	10
S_2	5	7	6	S_2	8	10	9
S_3	4	6	5	S_3	7	9	8

Table 8: Comparative results of different distributions.

Distribution	Transportation Cost	Expected Stochastic Cost	Total Cost (Numerator)	Revenue (Denominator)	Fractional Objective Value (Z)
Logarithmic	1240	620	1860	4960	0.375
Exponential	1260	600	1860	4960	0.375
Gamma	1225	640	1865	4960	0.376
Weibull	1205	670	1875	4960	0.378
Poisson	1250	580	1830	4960	0.369

A comparative analysis of the results obtained under different distributions is presented in Table 8. The comparison shows noticeable differences in transportation cost and expected shortage values across the distributions. Among the considered cases, the Poisson distribution provides a relatively efficient solution in terms of the fractional objective value, indicating better performance under discrete demand conditions. In contrast, other distributions exhibit higher variability in cost components due to differences in their structural characteristics. Overall, the numerical results demonstrate that the proposed model effectively captures the impact of uncertain demand and provides a flexible framework for optimal decision-making in transportation systems.

Conclusion:

This paper presented for a comprehensive stochastic fractional solid transportation model that incorporates multiple probability distributions to represent uncertain demand conditions. The integration of fractional programming and KKT optimality conditions enables an efficient solution framework for complex logistics systems. The comparative analysis reveals that different distributions significantly influence transportation cost and system efficiency, with the Poisson distribution demonstrating superior performance in discrete demand environments. The proposed model offers enhanced flexibility and practical applicability for decision-makers dealing with uncertainty in transportation planning. Future research may extend this framework to multi-objective, fuzzy, and dynamic transportation environments, as well as real-world case studies for validation.

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