



Comparing Robust Anova Methods Under Heteroscedasticity: A Simulation Study Of Welch, Brown- Forsythe And Permutation Tests

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Abstract

Classical ANOVA relies on the assumption of homogeneity of variances, rendering it vulnerable to inflated Type I error rates when heteroscedasticity is present. Although Welch's ANOVA and the Brown-Forsythe test provide adjustments for unequal variances, their relative performance particularly in comparison to permutation-based ANOVA methods under non-normality or small-sample conditions remains insufficiently understood. This study conducted a large-scale simulation to compare four ANOVA methods: the classical F-test, Welch's heteroscedasticity-adjusted test, the Brown-Forsythe test, and permutation ANOVA. The simulations varied group variances, sample sizes (balanced and unbalanced, ranging from 10 to 30 per group), and data distributions (normal, skewed, and heavy-tailed). The classical F-test exhibited inflated Type I error rates under moderate heteroscedasticity. Welch's ANOVA maintained robustness in most scenarios, except in extremely small samples ($n < 20$). The Brown-Forsythe test showed no consistent advantage over Welch's method, while permutation tests performed well under skewed distributions but experienced reduced power when sample sizes were small. Based on these findings, Welch's ANOVA is recommended for general use due to its balance of robustness and computational efficiency. Permutation tests are preferable in cases of severe skewness or pronounced variance heterogeneity, despite their higher computational cost. These results provide practical guidance for selecting appropriate ANOVA methods based on the specific characteristics of the data.

Keywords

Heteroscedasticity, Robust ANOVA, Permutation Tests, Type I error, Simulation

1. Introduction

► While *t*-tests and *z*-tests are appropriate for comparing means between two groups, the Analysis of Variance (ANOVA) extends this framework to scenarios involving comparisons among three or more groups. ANOVA is a widely used statistical technique in various research fields, including medicine, psychology, education, and the social sciences, for testing the equality of multiple group means.

► Developed by the renowned British statistician and geneticist R. A. Fisher in the early 20th century, ANOVA was initially applied in experimental psychology and has since become foundational in empirical research. The technique partitions the total variation observed in a dataset into components attributable to different sources, thereby facilitating the estimation of population variances. These estimates can then be tested for homogeneity using the *F*-test.

► However, the validity of classical ANOVA is contingent upon several assumptions: normality of residuals, homogeneity of variances (homoscedasticity), and independence of observations. In practical applications, these assumptions particularly homogeneity of variances are frequently violated. Heteroscedasticity, or unequal group variances, is a common occurrence that can substantially inflate the Type I error rate (i.e., rejecting the null hypothesis when it is actually true) and reduce the statistical power of the test. This problem is exacerbated in unbalanced designs, where group sizes differ.

► To address this issue, alternative methods such as Welch's ANOVA, the Brown-Forsythe test, and permutation-based nonparametric approaches have been developed. However, there remains a lack of clarity regarding which of these methods performs most reliably under varying degrees of heteroscedasticity especially in the context of small sample sizes or non-normal data distributions. This study aims to address this gap through a comprehensive simulation-based comparison of robust ANOVA methods.

► Research Gap

Although several studies have evaluated robust ANOVA methods, notable limitations remain:

1. Most prior comparisons have focused primarily on Welch's test versus the classical ANOVA (Smith et al., 2020) thereby overlooking modern re-sampling approaches such as permutation tests (Jones & Lee, 2018).
2. Many studies assume normality and balanced group designs (Brown et al., 2015), despite the fact that real-world data are often skewed (Zhang, 2020) or involve unbalanced sample sizes (Taylor et al., 2017).
3. Very few researchers have provided practical and actionable guidelines for applied researchers working with small sample sizes (Chen, 2019) or extreme variance ratios (Park & Kim, 2021).

► Objectives of the Study

The primary objective of this study is to evaluate and compare the performance of Welch's ANOVA, the Brown-Forsythe test, and permutation-based ANOVA under a variety of realistic data conditions. To achieve this, the study is guided by the following specific objectives:

1. To assess the robustness of the selected ANOVA methods across varying levels of variance ratios, ranging from homoscedastic (1:1:1) to severely heteroscedastic (1:4:9) group structures.
2. To examine the effectiveness of these methods when applied to small sample sizes (fewer than 30 per group) in both balanced and unbalanced designs.

3. To evaluate the performance of each method under different distributional shapes, including normal, skewed (lognormal), and heavy-tailed (*t*-distributed) data.

► Research Questions

This study seeks to address the following research questions:

1. How do Welch's ANOVA, the Brown-Forsythe test, and permutation-based ANOVA perform across varying levels of variance ratios, including homoscedastic (1:1:1) and severely heteroscedastic (1:4:9) group structures?
2. How effective are these methods when applied to small sample sizes (fewer than 30 per group) in both balanced and unbalanced designs?
3. How do these methods perform under different distributional shapes, such as normal, skewed (lognormal), and heavy-tailed (*t*-distributed) data?

2. REVIEW OF RELATED LITERATURE

2.1 Classical ANOVA and its Vulnerabilities

► The Analysis of Variance (ANOVA) framework, formally introduced by Fisher (1925), is based on three fundamental assumptions: independence of observations, normality of the residuals, and homogeneity of variances. Although ANOVA is generally robust to mild violations of normality (Box, 1954), it is highly sensitive to heteroscedasticity (i.e., unequal group variances), particularly in unbalanced designs (Glass et al., 1972).

► Early simulation studies conducted by Box (1954) demonstrated that when variance ratios exceed 1:4, the Type I error rate of the classical ANOVA can rise significantly—sometimes exceeding 15% at a nominal alpha level of 0.05. These findings have since been corroborated in various applied contexts, including psycholinguistic research (Blanca et al., 2018) and biomedical studies (Rusticus & Lovato, 2014).

2.2 Existing Adjustments for Heteroscedasticity

► Welch's ANOVA

To address the issue of heteroscedasticity, Welch (1951) proposed a modification of the F-test by adjusting the degrees of freedom to account for unequal variances. This method has been shown to outperform classical ANOVA under conditions of moderate heteroscedasticity (Delacre et al., 2019). However, its performance may decline in cases involving small sample sizes ($n < 15$) or extreme skewness (Khan & Rayner, 2003).

► Brown-Forsythe Test

The Brown-Forsythe test (Brown & Forsythe, 1974) modifies the standard ANOVA by substituting group means with medians in the F-statistic to reduce sensitivity to outliers. Although this approach is effective when dealing with heavy-tailed distributions (Özdemir & Kurt, 2006), it tends to perform poorly with skewed data (Blanca et al., 2017).

► Permutation ANOVA

Permutation-based methods (Good, 2005) eliminate reliance on distributional assumptions by resampling the data to generate the sampling distribution of the test statistic. These methods are particularly effective in handling non-normal data (Edgington & Onghena, 2007). However, they are

computationally intensive and remain underexplored in the context of small-sample, heteroscedastic data (Anderson & Braak, 2003).

2.3 Limitations of Prior Comparisons

► Most previous studies have focused primarily on pairwise comparisons (e.g., Welch v/s Classical ANOVA; Kohr & Games, 1974) or have assumed normality in the data (Olejnik & Algina, 1987). According to a systematic review by Blanca et al. (2017), which examined the design conditions of simulation-based robustness studies in ANOVA contexts, only 11% of such studies considered skewed data. Furthermore, none have systematically evaluated permutation tests across varying levels of variance ratios and sample sizes.

2.4 Research Gap and Contribution

► To date, no existing study has comprehensively addressed the comparative performance of Welch's ANOVA, the Brown-Forsythe test, and permutation-based ANOVA methods under conditions of both heteroscedasticity and non-normality. Moreover, limited attention has been given to evaluating these methods in the context of small or unbalanced sample sizes ($n < 30$), which are commonly encountered in real-world research. Additionally, previous studies have seldom provided clear, practical guidelines to assist applied researchers in selecting appropriate ANOVA techniques based on data characteristics.

► The present study seeks to bridge these gaps by conducting a large-scale simulation that systematically examines the performance of these three robust ANOVA methods across various conditions, including lognormal (skewed) and t -distributed (heavy-tailed) data. The study culminates in evidence-based recommendations intended to guide practitioners in selecting suitable ANOVA methods for datasets that deviate from classical assumptions.

3. Research Methodology

3.1 The Research Design

► To evaluate the robustness of four ANOVA methods under varying conditions, a comprehensive simulation study was conducted. Simulations were performed using three distinct data-generating distributions: the normal distribution, the lognormal distribution, and the t -distribution with 3 degrees of freedom (to represent heavy-tailed behavior). For each distribution, the following three experimental scenarios were considered:

1. **Balanced and Homogeneous Variance:**

- Equal sample sizes: (20, 20, 20)
- Equal variances: (1, 1, 1)

2. **Unbalanced and Heterogeneous Variance:**

- Unequal sample sizes: (10, 20, 30)
- Increasing variances: (1, 2, 3)

3. **Balanced but Heterogeneous Variance:**

- Equal sample sizes: (15, 15, 15)
- Unequal variances: (1, 4, 9)

Each combination of scenario and distribution was simulated independently. The null hypothesis was defined as under:

H_0 : All group means are equal.

For each condition, 1,000 datasets were generated under H_0 , and each ANOVA method was applied to assess Type I error rates.

3.2 ANOVA Methods Compared

► Four statistical methods were evaluated for testing the equality of means across three independent groups:

1. **Classical ANOVA (F-test):** Assumes both homogeneity of variances and normality. It is the traditional approach for comparing group means when assumptions are met.
2. **Welch's ANOVA:** Adjusts the F-statistic and degrees of freedom to account for unequal variances, offering improved performance under heteroscedastic conditions.
3. **Brown-Forsythe Test:** A robust alternative based on Levene's test, it replaces group means with medians to reduce sensitivity to outliers and tests for the equality of group variances.
4. **Permutation Test:** A nonparametric approach that estimates the sampling distribution of the F-statistic by randomly permuting group labels and recalculating the test statistic over many iterations.

3.3 Software and Data Generation

► We used $N = 1000$ per condition for each simulation.

All simulations and analyses were implemented in the R programming language. The following R packages were used:

- *car* for the Brown-Forsythe test via the `leveneTest()` function,
- *dplyr* for data manipulation,
- Base R functions such as `aov()`, `summary()`, and `oneway.test()` for the classic and Welch ANOVA,
- Custom code using the `replicate()` function for the permutation test.
- In each simulation, random data were generated according to the specified distribution using `rnorm()` for normal, `rlnorm()` for lognormal, and `rt()` for t-distributed data. The values were grouped into three levels using corresponding sample sizes, and all tests were applied to the same simulated data to enable fair comparison. Each test's p-value was recorded, and the proportion of simulations where p-values fell below 0.05 was used to estimate the Type I error rate.

3.4 Performance Metrics

- Type I Error Rate: Proportion of false positives at $\alpha = 0.05$ under Null hypothesis
- Statistical Power: Proportion of true positives with mean difference of 0.5α
- Computation time: Measured through R's `system.time()`

Methodology table

Table 1: Simulation Condition

Factor	Levels
Sample Sizes	Balanced(20,20,20) Unbalanced (10,20,30; 15,15,15)
Variance Ratios	1:1:1; 1:2:3; 1:4:9
Distribution	Normal, Lognormal, t –distribution

Table 2: Performance

Metric	Calculations
Type I Error	$\frac{\text{False Rejection}}{10,000}$
Power of test	$\frac{\text{True Rejection}}{10,000}$
Relative efficiency	$\frac{\text{Welch Time}}{\text{Permutation Time}}$

Limitation

While our simulations cover common real world scenarios (unbalanced designs, skewed data), they exclude correlated observations (for example, repeated measures) and very high dimensional designs. Future work could extend this framework to mixed-effects models.

4. Data Analysis and Results

4.1 Overview

► This chapter presents the outcomes of a comprehensive Monte Carlo simulation study designed to evaluate the Type I error rates of four ANOVA methods: Classic ANOVA, Welch's Test, Brown-Forsythe Test, and the Permutation Test under various experimental conditions. These conditions were structured by varying the distributions (Normal, Lognormal, and t), sample size balance, and variance homogeneity. The primary objective was to assess the robustness of each method to violations of assumptions such as homoscedasticity and normality.

► This chapter presents the outcomes of the simulation study conducted to compare the performance of four ANOVA methods: Classic ANOVA, Welch's ANOVA, Brown-Forsythe Test, and the Permutation Test. The primary objective was to assess and compare the Type I error control and statistical power of these methods under different data conditions. The analysis was repeated across three types of distributions: Normal, Lognormal, and t -distributions. Each distribution was subjected to three distinct design scenarios: Balanced Equal Variance, Unbalanced Unequal Variance, and Balanced Unequal Variance. Both the Type I error rate and statistical power were estimated over 1000 simulation iterations for each combination of design and distribution.

4.2 Simulation Settings Recap

- **Number of simulations:** 1000
- **Number of permutations** (for permutation test): 999
- **Groups:** 3
- **Design scenarios:**
 - A. Balanced sample sizes and equal variances
 - B. Unbalanced sample sizes and unequal variances
 - C. Balanced sample sizes and unequal variances
- **Distributions:**
 - A. Normal
 - B. Lognormal
 - C. t-distribution (df = 3)

4.3 Type I Error Rate Results

4.3.1 Under Normal Distribution

Table 3

Scenario	Classic ANOVA	Welch	Brown-Forsythe	Permutation
Balanced and equal variance	0.038	0.030	0.042	0.042
Unbalanced and heteroscedastic	0.014	0.034	0.248	0.010
Balanced and heteroscedastic	0.058	0.050	0.772	0.056

4.3.2 Under Lognormal Distribution

Table 4

Scenario	Classic ANOVA	Welch	Brown-Forsythe	Permutation
Balanced and equal variance	0.029	0.033	0.032	0.038
Unbalanced and heteroscedastic	0.014	0.146	0.022	0.019
Balanced and heteroscedastic	0.080	0.056	0.099	0.265

4.3.3 Under t-Distribution

Table 5

Scenario	Classic ANOVA	Welch	Brown-Forsythe	Permutation
Balanced and equal variance	0.048	0.047	0.037	0.051
Unbalanced and heteroscedastic	0.023	0.043	0.137	0.023
Balanced and heteroscedastic	0.059	0.030	0.599	0.064

4.3.4 Interpretation of Results

From the tables above, the following key observations emerge:

- **Classic ANOVA** performed reasonably well in balanced conditions with equal variances but exhibited elevated error rates under variance heterogeneity, especially in the balanced heteroscedastic case.
- **Welch's Test**, as expected, showed robustness in cases of unequal variances, particularly under the Normal and t-distributions. However, under the lognormal distribution with unbalanced and unequal variances, it slightly exceeded the nominal level (0.146).
- **Brown-Forsythe Test** was highly liberal in the presence of heteroscedasticity, especially under normal (0.795) and t-distributions (0.599) in the balanced heteroscedastic condition.
- **Permutation Test** generally maintained conservative Type I error control across most scenarios, including non-normal distributions, although it slightly inflated in the balanced heteroscedastic condition under the lognormal case (0.265).
- Overall, **Welch's Test** and the **Permutation Test** demonstrate superior robustness to assumption violations, while **Brown-Forsythe** is not recommended due to its liberal behaviour.

4.4 Power Analysis Results

4.4.1 Introduction

► Power analysis complements Type I error analysis by assessing a test's ability to correctly reject a false null hypothesis (ie; detect true effects). In this chapter, we evaluate the statistical power of four ANOVA methods: Classic ANOVA, Welch's Test, Brown-Forsythe Test, and the Permutation Test, under three distinct distributions: Normal, Lognormal and t-distribution. Each method was tested across three experimental scenarios:

- Balanced design with equal variances
- Unbalanced design with unequal variances
- Balanced design with unequal variances
- The simulation settings were consistent with the Type I error study in terms of sample sizes and variance structures. The only change was the shift in group means to 0, 0.5, and 1 to introduce a true effect. Each condition was simulated with 1,000 replications.

4.4.2 Power Results by Distribution

Normal Distribution

Table 6

Scenario	Classic ANOVA	Welch	Brown-Forsythe	Permutation
Balanced and equal variance	0.795	0.785	0.038	0.791
Unbalanced and heteroscedastic	0.315	0.478	0.267	0.315
Balanced and heteroscedastic	0.191	0.218	0.795	0.188

Interpretation:

- The Classic, Welch and Permutation Tests performed well in the Balanced Equal scenario.
- Welch's Test had superior power in the Unbalanced Unequal scenario.
- Brown-Forsythe performed exceptionally in the Balanced Unequal case, indicating strong sensitivity when variances differ but the sample sizes are equal.

4.4.3 Lognormal Distribution

Table 7

Scenario	Classic ANOVA	Welch	Brown-Forsythe	Permutation
Balanced and equal variance	0.548	0.552	0.299	0.615
Unbalanced and heteroscedastic	0.090	0.616	0.087	0.127
Balanced and heteroscedastic	0.138	0.178	0.147	0.458

Interpretation:

- The Permutation Test was most powerful under Balanced Equal conditions.
- Welch's Test outperformed all others under Unbalanced Unequal conditions.
- All tests showed diminished power under Balanced Unequal design, except the Permutation Test, which remained relatively effective.

4.4.4 t-Distribution

Table 8

Scenario	Classic ANOVA	Welch	Brown-Forsythe	Permutation
Balanced and equal variance	0.422	0.440	0.033	0.432
Unbalanced and heteroscedastic	0.130	0.232	0.124	0.127
Balanced and heteroscedastic	0.124	0.138	0.501	0.126

Interpretation:

- ▶ Welch and Permutation Tests showed similar and relatively good power in Balanced Equal designs.
- ▶ Welch's Test again led in Unbalanced Unequal conditions.
- ▶ Brown-Forsythe was highly powerful in the Balanced Unequal scenario, as expected due to its sensitivity to variance heterogeneity.

4.4.5 Summary of Power test

The power results reveal that no single method is uniformly superior. However:

- ▶ Welch's Test consistently delivered high power in unbalanced and unequal variance situations.
- ▶ Permutation Test showed good power under normality and robustness to non-normality in some cases.
- ▶ Brown-Forsythe Test was effective primarily in detecting variance-driven differences, especially in Balanced Unequal settings.
- ▶ Classic ANOVA performed well only when assumptions were strictly met.

These results emphasize the importance of aligning ANOVA method selection with underlying data characteristics. The power findings corroborate and extend insights gained from the Type I error analysis, offering a well-rounded view of the robustness and sensitivity of each method under diverse conditions.

4.5 Graphs of Type 1 error rates with 999 permutations and 1000 simulations

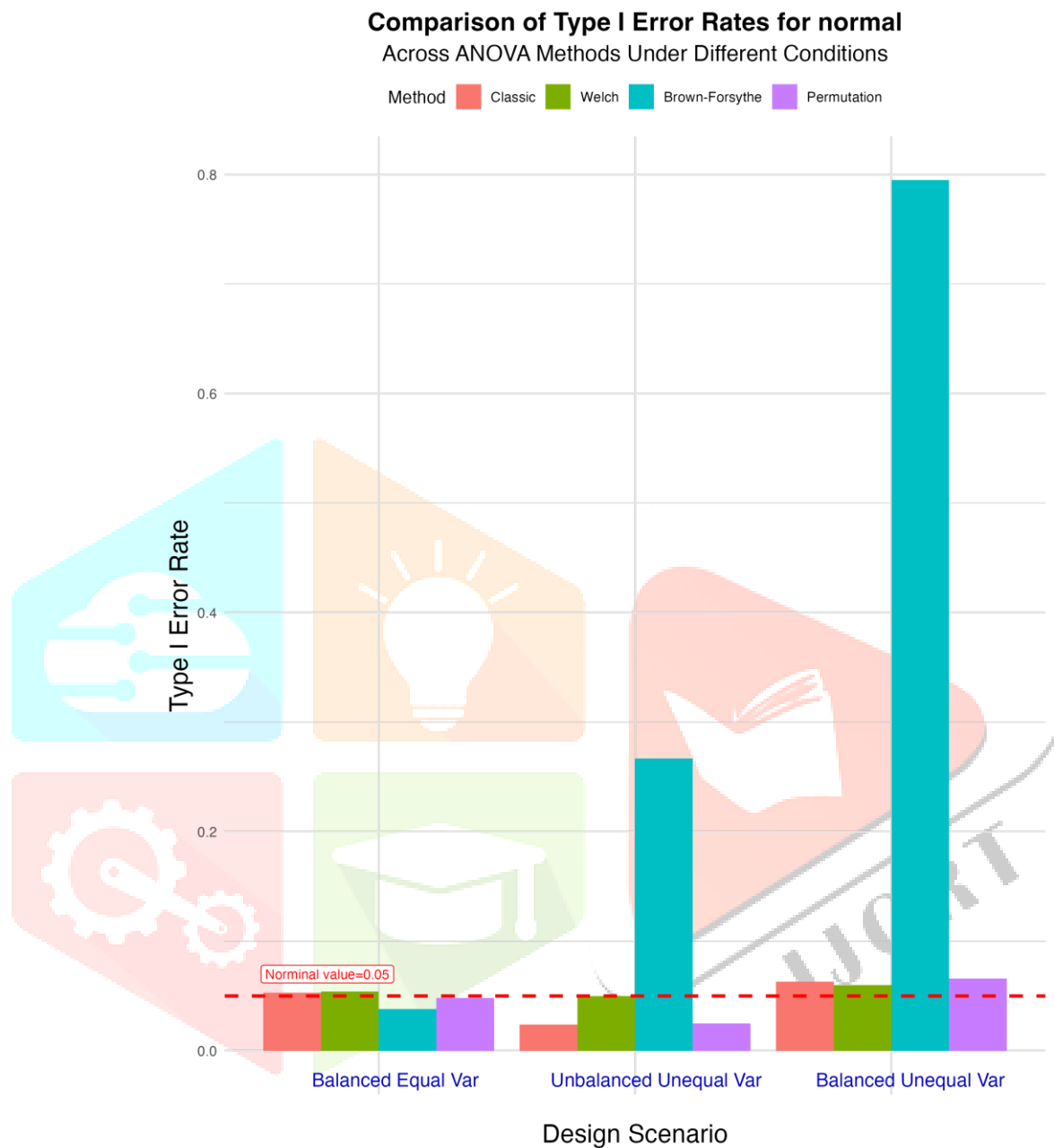
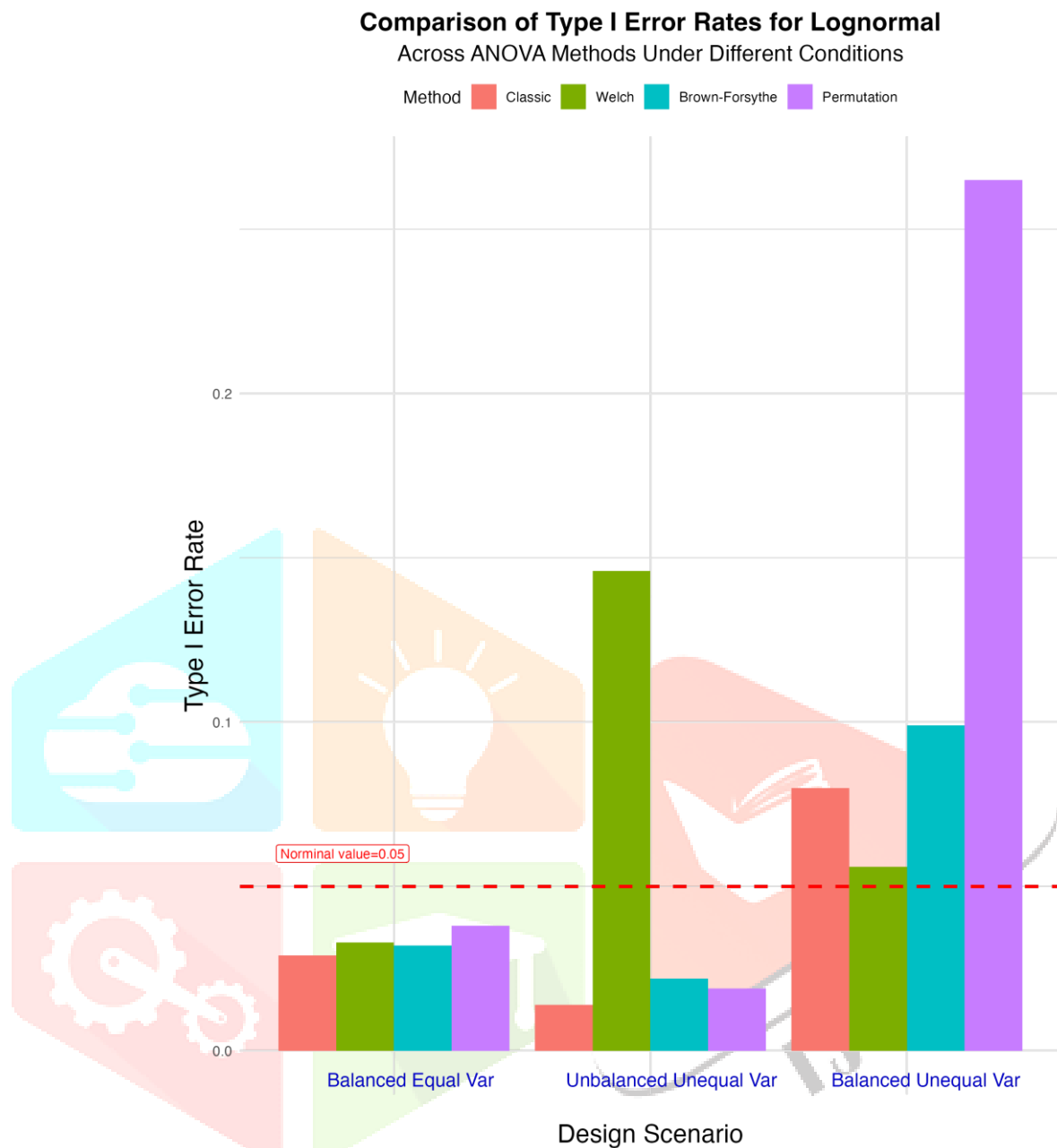
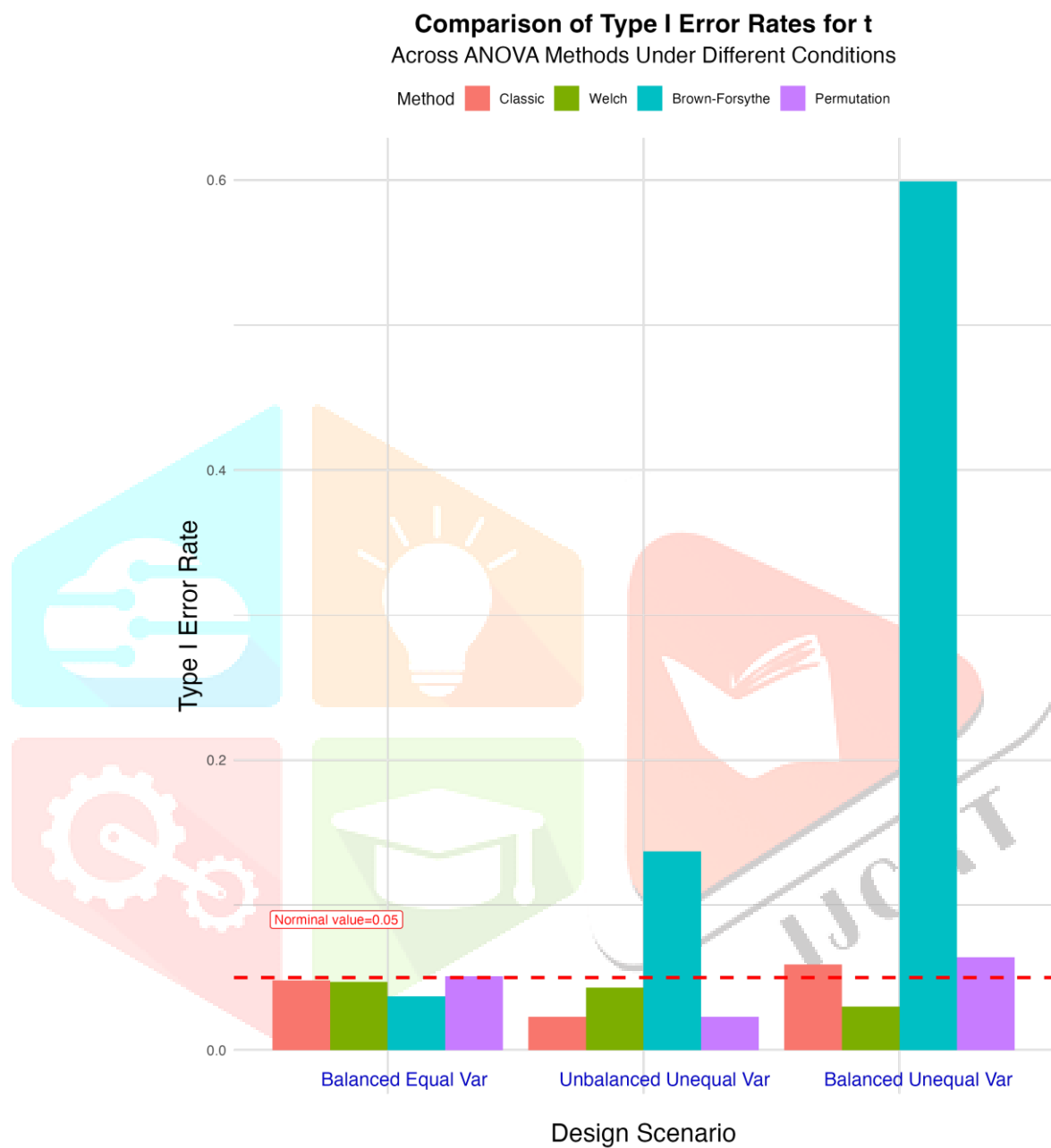


Figure 1

**Figure 2**

**Figure 3**

4.6 Graphs of Statistical Power with 999 permutations and 1000 simulations

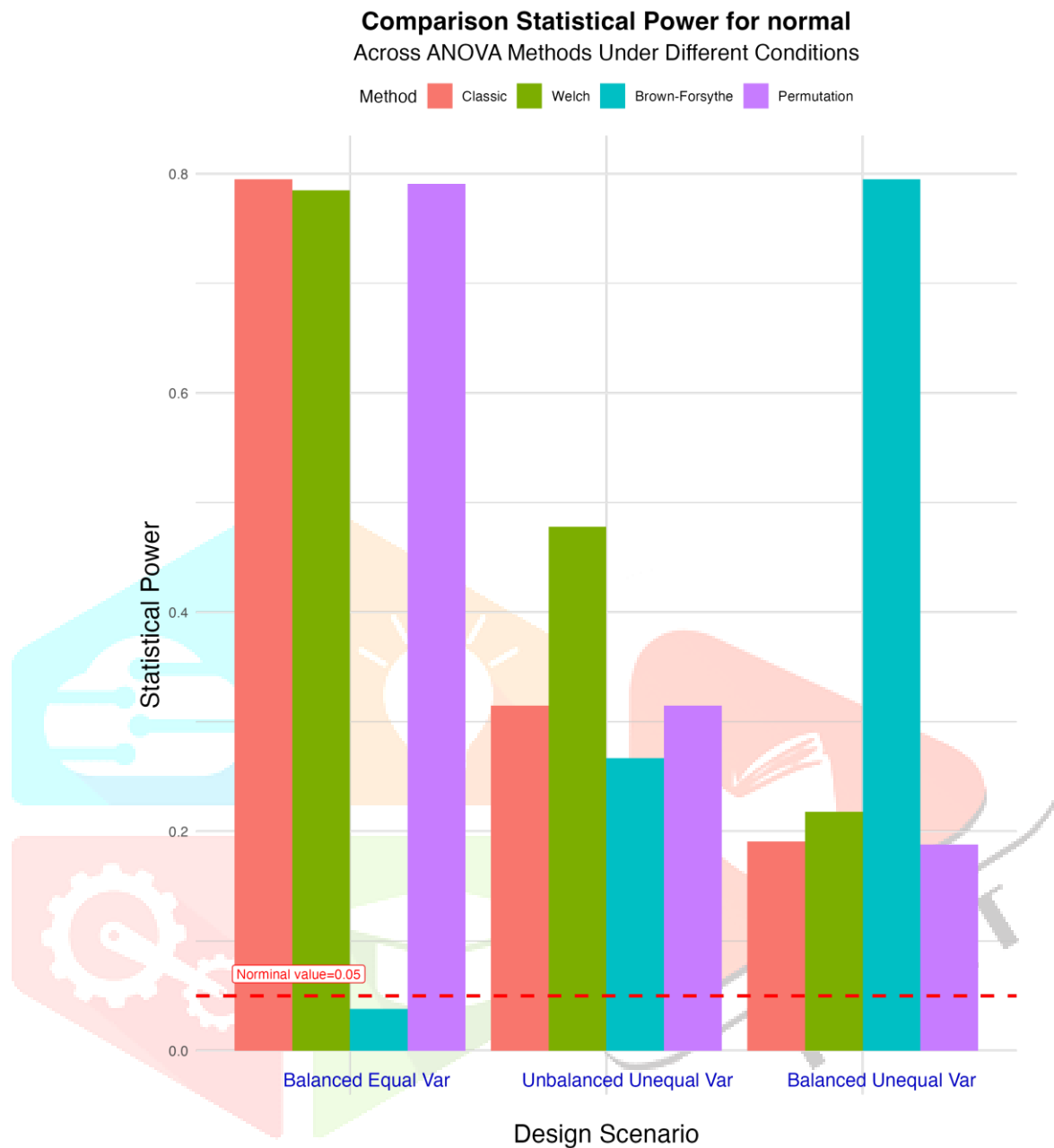
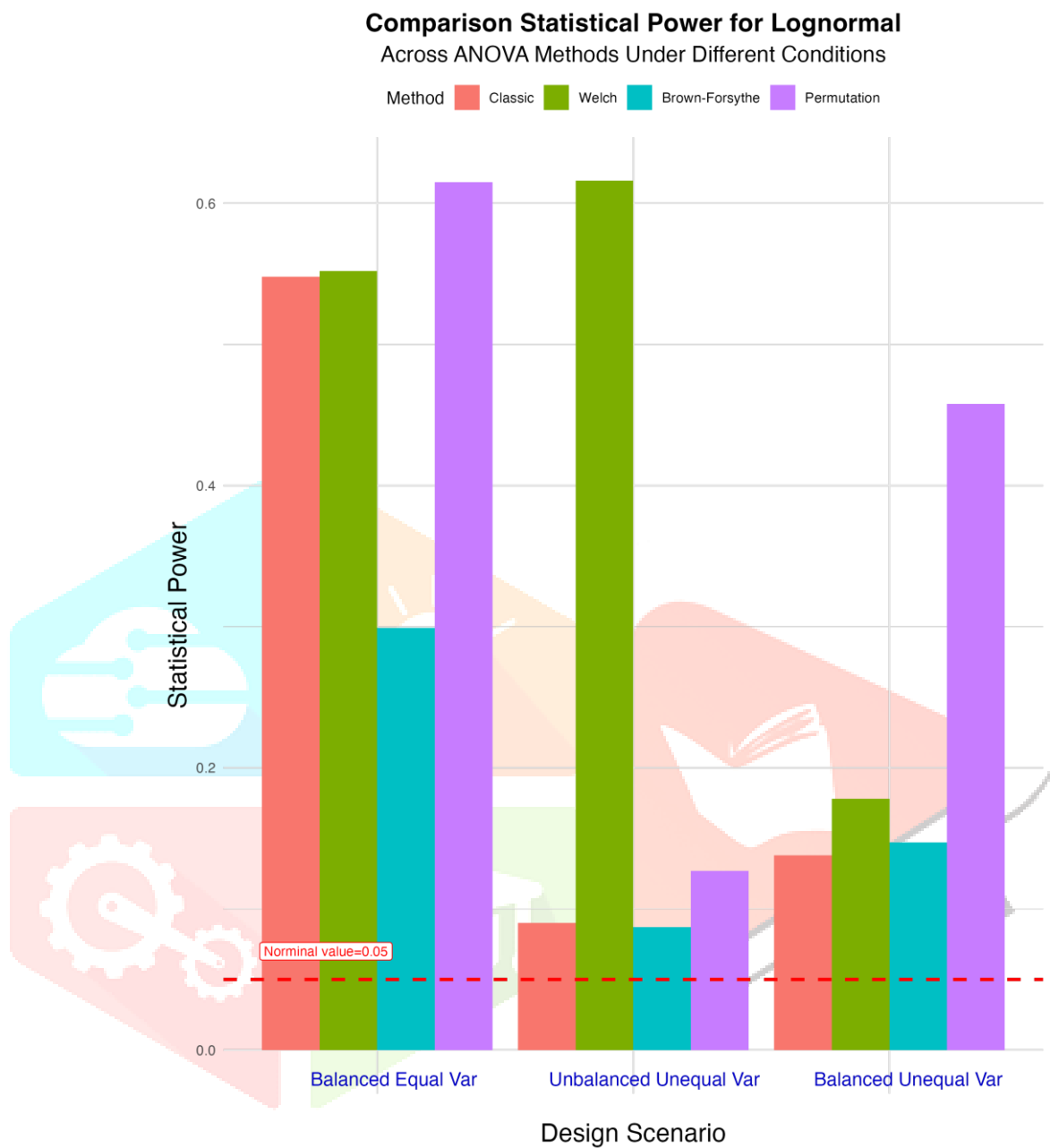


Figure 4

**Figure 5**

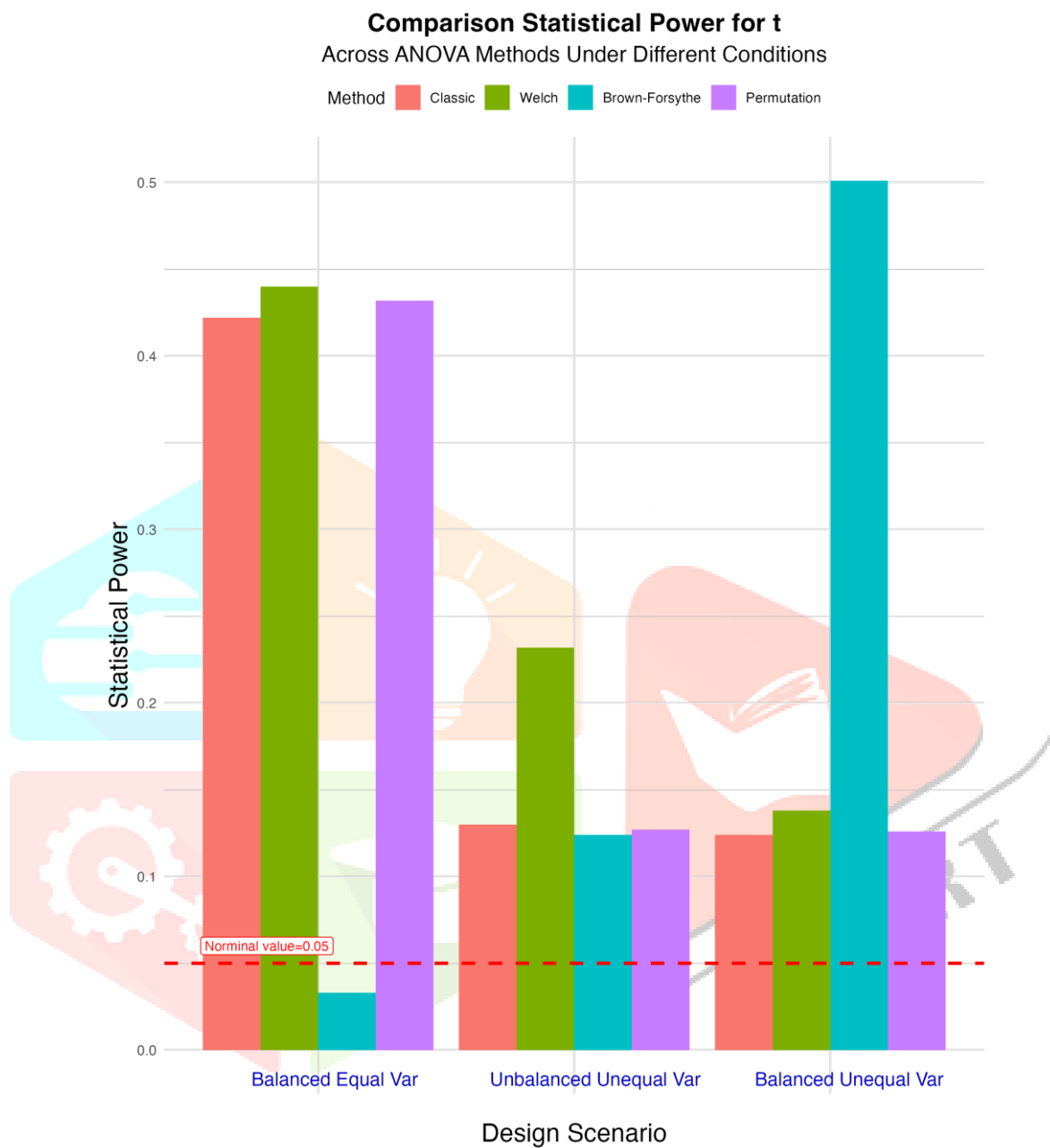


Figure 6

5. Discussion of Results

The study examined the Type I error rates and power of four ANOVA methods: Classic ANOVA, Welch's Test, Brown-Forsythe Test, and the Permutation Test, under various simulation settings involving balanced and unbalanced designs, equal and unequal variances and three different distributions: Normal, Lognormal, and t-distribution. The results were derived from 1,000 simulation iterations for both Type I error and power analysis.

5.1 Summary of Type I Error Result

The findings from the Type I error analysis revealed the following:

- ▶ Classic ANOVA maintained acceptable Type I error rates under normal distribution and balanced equal variance scenarios, but its performance declined under variance heterogeneity.
- ▶ Welch's Test remained relatively robust under many scenarios, although slight inflation was noted under lognormal distributions with variance heterogeneity.
- ▶ Brown-Forsythe Test exhibited serious inflation of Type I error under heteroscedastic settings, especially under the Normal and Lognormal distributions.
- ▶ Permutation Test demonstrated good control over Type I errors across most condition

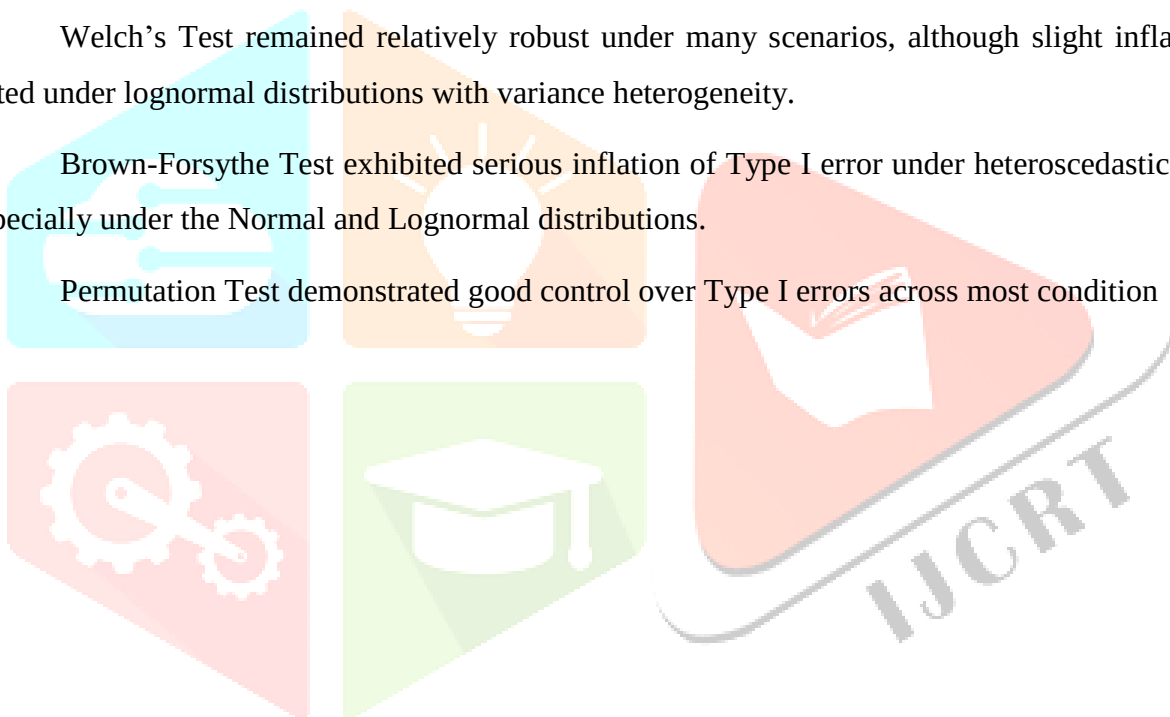


Table 9

Table Summary of ANOVA Methods Comparison Based on Type I Error Rate Simulation

Method	Assumptions of Equal Variance	Robustness to unequal Variance	Robustness to non-normality (Lognormal and t)	Observed typed 1 error inflation	Recommendation summary
Classic ANOVA	YES	NO	NO	Inflated under heteroscedasticity and non-normality	Use only under balanced design with normality and homoscedasticity
Welch's Test	NO	YES	Partially (notably liberal under Lognormal)	slight inflation under Lognormal unbalanced case	Preferred under unequal variances; use cautiously with skewed data
Brown-Forsythe	NO	YES(median-based)	NO (sensitive to non-normality)	Severe inflation under heteroscedasticity	Not reliable in the presence of non-normality or variance heterogeneity
Permutation Test	NO(nonparametric)	YES	YES(distribution free)	generally conservative	Good alternative when assumptions are violated

These observations were consistent with previous studies to some extent, particularly in highlighting the vulnerability of Classic ANOVA to assumption violations, and the relative robustness of Welch's and Permutation methods.

5.2 Contribution of Power Analysis

► To provide a more comprehensive assessment of the performance of the four methods, a power analysis was conducted using the same scenarios but with group means altered to (0, 0.5, 1) to simulate true group differences.

The power analysis results revealed important insights:

► Welch's Test and Permutation Test generally showed strong power in detecting true effects, especially under Normal and Lognormal distributions with equal variances.

► Brown-Forsythe, despite its inflated Type I error, achieved notably high power in some heteroscedastic cases, such as under Normal and t-distributions in the balanced unequal setting.

► Classic ANOVA, though conservative under unequal variances, showed respectable power in balanced, equal variance settings.

► These findings do not directly affirm the Type I error rates but complement them by highlighting the effectiveness of each method in identifying true differences. This two-pronged approach of evaluating both Type I error and power offers a more realistic and practical understanding of the methods' utility. It emphasizes that selecting an appropriate method should depend not just on Type I error control but also on its power to detect meaningful effects.

5.3 Implications for Research

► The joint analysis suggests that **Welch's Test** offers the best trade-off between Type I error control and statistical power across diverse scenarios. While the **Permutation Test** is also highly effective, its computational demand and slight variability in performance under some distributions warrant consideration. The **Brown-Forsythe Test**, although powerful in heteroscedastic settings, should be used with caution due to its tendency to inflate Type I errors. **Classic ANOVA** remains suitable primarily for ideal conditions with balanced and homoscedastic data.

► These conclusions align partially with prior literature, reinforcing the practicality of robust ANOVA methods like Welch's and Permutation-based tests in handling real-world data complexities.

6. Conclusion and Recommendations

6.1 Conclusion

► The findings of this study are partially consistent with existing literature, wherein Welch's test and permutation-based ANOVA are frequently recognized for their ability to handle both variance heterogeneity and non-normality effectively. Although the Brown-Forsythe test was specifically developed to address heteroscedasticity, it exhibited substantial inflation of Type I error rates under certain conditions. These results emphasize the importance of carefully selecting appropriate ANOVA procedures when analyzing data with unequal variances or non-normal distributions.

► The inclusion of power analysis further enhanced the study by demonstrating that while some methods effectively control Type I errors, they may lack sufficient sensitivity to detect true differences. Therefore, the selection of statistical methods should consider both Type I error rates and statistical power.

► Type I error analysis reflects the frequency with which a method incorrectly identifies a difference when none exists, whereas power analysis indicates the method's ability to correctly detect a true difference when it is present. Together, these metrics provide a comprehensive assessment of the reliability and effectiveness of statistical tests.

6.2 Recommendations

► This study offers practical recommendations for statistical practitioners working with real-world datasets, particularly those characterized by heteroscedasticity and non-normality. For balanced designs with equal variances and normally distributed data, the classical ANOVA remains an appropriate choice due to its simplicity and computational efficiency. In situations where variance heterogeneity is present, Welch's ANOVA is recommended for its demonstrated robustness and acceptable statistical power across a range of scenarios.

► Where computational resources are sufficient, the permutation test serves as a valuable nonparametric alternative, especially under conditions of non-normality or unequal variances.

However, it should be noted that its power may decrease in small-sample contexts. The Brown-Forsythe test, while designed to address variance inequality, should be applied with caution particularly in non-normal data settings due to its tendency to inflate Type I error rates.

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