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On Fine $\mathcal{G}_\alpha$ -Quotient Mappings

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Abstract:

The aim of this paper is to introduce fine $\mathcal{G}_\alpha$ -quotient map using fine $\mathcal{G}_\alpha$ -closed sets and studied their basic properties. We also study the relation between weak and strong form of fine $\mathcal{G}_\alpha$ -quotient maps. We also derive the relation between strongly fine $\mathcal{G}_\alpha$ -continuous map and fine $\mathcal{G}_\alpha$ -quotient maps.

Keywords: #gs-closed sets, #gs-open sets, fine $\mathcal{G}_\alpha$ -closed and fine $\mathcal{G}_\alpha$ -open sets.

AMS Subject Classification: 54XX.

1. Introduction

Powar P. L. and Rajak K. [5], have investigated a special case of generalized topological space called fine topological space. In this space, they have defined a new class of open sets namely fine-open sets which contains all α –open sets, β –open sets, semi-open sets, pre-open sets, regular open sets etc.. By using these fine-open sets they have define fine-irresolute mappings which include pre-continuous functions, semi-continuous function, α –continuous function, β –continuous functions, α –irresolute functions, β –irresolute functions, etc (cf. [5]-[12]).

In this paper, we have introduced fine $\mathcal{G}_\alpha$ -quotient map using fine $\mathcal{G}_\alpha$ -closed sets and study their basic properties. We also study the relation between weak and strong form of fine $\mathcal{G}_\alpha$ -quotient maps. We also derive the relation between strongly fine $\mathcal{G}_\alpha$ -continuous map and fine $\mathcal{G}_\alpha$ -quotient maps. Examples are given to illustrate the results.

2. Preliminaries

Throughout this paper, (X, τ, τ_f) and (Y, σ, σ_f) means fine topological spaces on which no separation axioms are assumed. For a subset A of a space X the closure and interior of A with respect to τ are denoted by $f_{cl}(A)$ and $f_{int}(A)$. We use the following definitions:

Definition 2.1 A subset A of a fine topological space (X, τ, τ_f) is called

- (i) ω –closed set [16] ($= \check{g}$ –closed) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is *semi* –open in (X, τ)

Definition 2.2 A function $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called

- (i) $\tilde{g}_\alpha$ -continuous [14] if $f^{-1}(V)$ is a $\tilde{g}_\alpha$ –closed set of (X, τ, τ_f) for each closed set of (Y, σ, σ_f) ,

Definition 2.3 Let $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ be a map. Then f is said to be

- (i)a $\tilde{g}_\alpha$ -irresolute [14] if $f^{-1}(V)$ is $\tilde{g}_\alpha$ -open in (X, τ, τ_f) for each $\tilde{g}_\alpha$ -open set V of (Y, σ, σ_f) ,

Definition 2.4 A surjective map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is said to be

- (i)a quotient map [4], provided a subset U of (Y, σ, σ_f) is open in (Y, σ, σ_f) if and only if $f^{-1}(U)$ is open in (X, τ, τ_f) ,

Definition 2.5 Let (X, τ, τ_f) be a fine topological space we define

$\tau(A_\alpha) = \tau_\alpha(\text{say}) = \{G_\alpha(\neq X) : G_\alpha \cap A_\alpha = \phi, \text{ for } A_\alpha \in \tau \text{ and } A_\alpha = \phi, X, \text{ for some } \alpha \in J, \text{ where } J \text{ is the index set.}\}$

Now, we define

$$\tau_f = \{\phi, X, \cup_{\{\alpha \in J\}} \{\tau_\alpha\}\}$$

The above collection τ_f of subsets of X is called the fine collection of subsets of X and (X, τ, τ_f) is said to be the fine space X generated by the topology τ on X (cf. [5]).

Definition 2.6 A subset U of a fine space X is said to be a fine-open set of X , if U belongs to the collection τ_f and the complement of every fine-open sets of X is called the fine-closed sets of X and we denote the collection by F_f (cf. [5]).

Definition 2.7 Let A be a subset of a fine space X , we say that a point $x \in X$ is a fine limit point of A if every fine-open set of X containing x must contains atleast one point of A other than x (cf. [5]).

Definition 2.8 Let A be the subset of a fine space X , the fine interior of A is defined as the union of all fine-open sets contained in the set A i.e. the largest fine-open set contained in the set A and is denoted by f_{Int} (cf. [5]).

Definition 2.9 Let A be the subset of a fine space X , the fine closure of A is defined as the intersection of all fine-closed sets containing the set A i.e. the smallest fine-closed set containing the set A and is denoted by f_{cl} (cf. [5]).

Definition 2.10 A function $f : (X, \tau, \tau_f) \rightarrow (Y, \tau', \tau'_f)$ is called fine-irresolute (or f -irresolute) if $f^{-1}(V)$ is fine-open in X for every fine-open set V of Y (cf. [5]).

Definition 2.11 A fine-open set S of a fine topological space (X, τ, τ_f) (cf. [5]) is called

1. αf –open if S is α –open subset of a topological space (X, τ) .
2. βf –open if S is β –open subset of a topological space (X, τ) .
3. pf –open if S is pre-open subset of a topological space (X, τ) .
4. sf –open if S is semi-open subset of a topological space (X, τ) .

Definition 2.12 A function $f: (X, \tau, \tau_f) \rightarrow (Y, \tau', \tau'_f)$ is said to be αf -irresolute if $f^{-1}(V)$ is αf -open in X for every αf -open set V of Y (cf. [5]).

Definition 2.13 A function $f: (X, \tau, \tau_f) \rightarrow (Y, \tau', \tau'_f)$ is said to be βf -irresolute if $f^{-1}(V)$ is βf -open in X for every βf -open set V of Y (cf. [5]).

Definition 2.14 A function $f: (X, \tau, \tau_f) \rightarrow (Y, \tau', \tau'_f)$ is said to be pf -irresolute if $f^{-1}(V)$ is pf -open in X for every pf -open set V of Y (cf. [5]).

Definition 2.15 A function $f: (X, \tau, \tau_f) \rightarrow (Y, \tau', \tau'_f)$ is said to be sf -irresolute if $f^{-1}(V)$ is sf -open in X for every sf -open set V of Y (cf. [5]).

Definition 2.16 Let X be a non-empty set and τ be a topology on X . Let A be a subset of X , then A is said to be $f\alpha$ -open if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$. The collection of all $f\alpha$ -open sets of X is again a topology. We denote this collection by $\tau_{f\alpha}$ i.e $\tau_{f\alpha}$ is the topology of $f\alpha$ -open sets of X . (cf. [5]).

3. $f\tilde{g}_\alpha$ -continuous functions

In this section we have defined $f\tilde{g}_\alpha$ -continuous maps in fine topological space.

Definition 3.1 A subset A of a fine topological space (X, τ, τ_f) is called

(ii) $f\omega$ -closed set ($= \tilde{f}g$ -closed) if $f_{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is *semi*-open in (X, τ, τ_f)

Example 3.2 Let X be a set and τ be a topology on X and τ_f be a fine topology on X

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Fine semi open sets $= \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$, The collection of $f\omega$ -closed sets in (X, τ, τ_f) is $\{\phi, X, \{b\}, \{c\}, \{b, c\}\}$.

(ii) $a * fg$ -closed set [9] if $f_{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $f\omega$ -open in (X, τ, τ_f) **Example 3.3**

Let X be a set and τ be a topology on X and τ_f be a fine topology on X

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

$f\omega$ -closed sets $= \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$ and $f\omega$ -open sets $= \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

The collection of $*fg$ -closed sets in (X, τ, τ_f) is $\{\phi, X, \{b\}, \{c\}, \{b, c\}\}$.

(iii) $a \# fg$ -semi-closed set (briefly $\#fgs$ -closed) [10] if $f_{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $*fg$ -open in (X, τ, τ_f) .

Example 3.4 Let X be a set and τ be a topology on X and τ_f be a fine topology on X

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

$*fg$ -closed sets $= \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$ and $*fg$ -open sets $= \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Fine semi open sets $= \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

The collection of $\#fg$ -semi-closed sets in (X, τ, τ_f) is $\{\phi, X, \{b\}, \{c\}, \{b, c\}\}$.

(iv) $a f\tilde{g}$ -closed set [5] if $f_{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\#fgs$ -open in (X, τ, τ_f) and **Example**

3.5 Let X be a set and τ be a topology on X and τ_f be a fine topology on X

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

$\#fg$ –semi-closed sets $= \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$ and $\#fg$ –semi-open sets $= \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$,

The collection of $f\tilde{g}$ –closed sets in (X, τ, τ_f) is $\{\phi, X, \{b\}, \{c\}, \{b, c\}\}$.

(v) a $f\tilde{g}_\alpha$ -closed set [2] if $f\alpha_{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\#fgs$ –open in (X, τ, τ_f) . **Example**

3.6 Let X be a set and τ be a topology on X and τ_f be a fine topology on X

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

$\#fg$ –semi-open sets $= \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$,

$f\alpha$ -closed sets $= \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$, The collection of $f\tilde{g}_\alpha$ -closed sets in (X, τ, τ_f) is

$\{\phi, X, \{b\}, \{c\}, \{b, c\}\}$.

The complement of $*fg$ –closed (resp $f\omega$ –closed, $\#fgs$ –closed, $f\tilde{g}$ –closed and $f\tilde{g}_\alpha$ –closed) set is said to $*fg$ –open (resp $f\omega$ –open, $\#fgs$ –open, $f\tilde{g}$ –open and $f\tilde{g}_\alpha$ –open).

Definition 3.7 A function $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is called

(i) $f\tilde{g}_\alpha$ -continuous if $f^{-1}(V)$ is a $f\tilde{g}_\alpha$ –closed set of (X, τ, τ_f) for each closed set of (Y, σ, σ_f) ,

Example 3.8 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y = \{1, 2, 3\}$ and $\sigma = \{\phi, Y, \{1\}\}$ and $\sigma_f = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

$\tilde{G}_{\alpha fcl}(X) = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$,

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a $f\tilde{g}_\alpha$ -continuous.

(ii) strongly $f\tilde{g}_\alpha$ -continuous [6] if $f^{-1}(V)$ is a closed set of (X, τ, τ_f) for each $f\tilde{g}_\alpha$ -closed set V of (Y, σ, σ_f) .

Example 3.9 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y = \{1, 2, 3\}$ and $\sigma = \{\phi, Y, \{1\}\}$ and $\sigma_f = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

$\tilde{G}_{\alpha fcl}(X) = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$, and $\tilde{G}_{\alpha fcl}(Y) = \{\phi, Y, \{2\}, \{3\}, \{2, 3\}\}$

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a strongly

$f\tilde{g}_\alpha$ -continuous.

Definition 3.10 Let $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ be a map. Then f is said to be

(i) a $f\tilde{g}_\alpha$ -irresolute if $f^{-1}(V)$ is $f\tilde{g}_\alpha$ -open in (X, τ, τ_f) for each $f\tilde{g}_\alpha$ -open set V of (Y, σ, σ_f) ,

(ii) a $f\tilde{g}$ -irresolute [3] if $f^{-1}(V)$ is $f\tilde{g}$ -open in (X, τ, τ_f) for each $f\tilde{g}$ -open set V of (Y, σ, σ_f) .

Example 3.11 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y = \{1, 2, 3\}$ and $\sigma = \{\phi, Y, \{1\}\}$ and $\sigma_f = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

$\tilde{G}_\alpha f_{cl}(X) = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$, and $\tilde{G}_\alpha O(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$,

$\tilde{G}_\alpha f_{cl}(Y) = \{\phi, Y, \{2\}, \{3\}, \{2, 3\}\}$ and $\tilde{G}_\alpha O(Y) = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a strongly

$f\tilde{G}_\alpha$ -irresolute.

Example 3.12 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y = \{1, 2, 3\}$ and $\sigma = \{\phi, Y, \{1\}\}$ and $\sigma_f = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

$f\tilde{g}$ -closed sets of $(X, \tau, \tau_f) = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$, and

$f\tilde{g}$ -open sets of $(X, \tau, \tau_f) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$,

$f\tilde{g}$ -closed sets of $(Y, \sigma, \sigma_f) = \{\phi, Y, \{2\}, \{3\}, \{2, 3\}\}$ and

$f\tilde{g}$ -open sets of $(Y, \sigma, \sigma_f) = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a strongly

$f\tilde{g}$ -irresolute.

Definition 3.15 A surjective map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is said to be

- (i) an $f\alpha$ -quotient map if f is $f\alpha$ -continuous and $f^{-1}(V)$ is open in (X, τ, τ_f) implies V is an $f\alpha$ -open set in (Y, σ, σ_f) and
- (ii) an $f\alpha^*$ -quotient map if f is $f\alpha$ -irresolute and $f^{-1}(V)$ is an $f\alpha$ -open set in (X, τ, τ_f) implies V is an open set in (Y, σ, σ_f) .

Example 3.16 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y = \{1, 2, 3\}$ and $\sigma = \{\phi, Y, \{1\}\}$ and $\sigma_f = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

$f\alpha$ -closed sets of $(X, \tau, \tau_f) = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$, and

$f\alpha$ -open sets of $(X, \tau, \tau_f) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$,

$f\alpha$ -closed sets of $(Y, \sigma, \sigma_f) = \{\phi, Y, \{2\}, \{3\}, \{2, 3\}\}$, and

$f\alpha$ -open sets of $(Y, \sigma, \sigma_f) = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a $f\alpha$ -quotient map.

Example 3.17 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y = \{1, 2, 3\}$ and $\sigma = \{\phi, Y, \{1\}\}$ and $\sigma_f = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

$f\alpha$ -closed sets of $(X, \tau, \tau_f) = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$

$f\alpha$ -open sets of $(X, \tau, \tau_f) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

$f\alpha$ -closed sets of $(Y, \sigma, \sigma_f) = \{\phi, Y, \{2\}, \{3\}, \{2, 3\}\}$

$f\alpha$ -open sets of $(Y, \sigma, \sigma_f) = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a $f\alpha^*$ - quotient map.

Definition 3.18 A subset A of a fine topological space (X, τ, τ_f) is called

(i) $f\omega$ -closed set ($= \widehat{f\tilde{g}}$ -closed) if $f_{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is fine semi-open in (X, τ, τ_f)

(ii) a $*fg$ -closed set if $f_{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $f\omega$ -open in (X, τ, τ_f) ,

(iii) a $\#fg$ -semi-closed set (briefly $\#fgs$ -closed) [10] if $f_{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $*fg$ -open in (X, τ, τ_f) .

(iv) a $f\tilde{g}$ -closed set if $f_{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\#fgs$ -open in (X, τ, τ_f) and

(v) a $f\tilde{g}_\alpha$ -closed set if $f_\alpha - Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\#fgs$ -open in (X, τ, τ_f) .

The complement of $*fg$ -closed (resp $f\omega$ -closed, $\#fgs$ -closed, $f\tilde{g}$ -closed and $f\tilde{g}_\alpha$ -closed) set is said to $*fg$ -open (resp $f\omega$ -open, $\#fgs$ -open, $f\tilde{g}$ -open and $f\tilde{g}_\alpha$ -open).

4. $f\tilde{g}_\alpha$ -quotient maps

In this section we have defined $f\tilde{g}_\alpha$ -quotient maps in fine topological space.

Definition 4.1 A surjective map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is said to be a $f\tilde{g}_\alpha$ -quotient map if f is $f\tilde{g}_\alpha$ -continuous and $f^{-1}(V)$ is open in (X, τ, τ_f) implies V is a $f\tilde{g}_\alpha$ -open set in (Y, σ, σ_f) . **Example-** Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y = \{1, 2, 3\}$ and $\sigma = \{\phi, Y, \{1\}\}$ and $\sigma_f = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a $f\tilde{g}_\alpha$ -quotient map.

Definition 4.2 A map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is said to be $f\tilde{g}_\alpha$ -open if $f(U)$ is $f\tilde{g}_\alpha$ -open in (Y, σ, σ_f) for each open set U in (X, τ, τ_f) .

Example- Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y = \{1, 2, 3\}$ and $\sigma = \{\phi, Y, \{1\}\}$ and $\sigma_f = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

$$\tilde{G}_\alpha O(Y) = \{ \phi, Y, \{1\}, \{1,2\}, \{1,3\} \},$$

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a $f\tilde{g}_\alpha$ -open.

Definition 4.3 A map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is said to be strongly $f\tilde{g}_\alpha$ -open if $f(U)$ is $f\tilde{g}_\alpha$ -open in (Y, σ, σ_f) for each $f\tilde{g}_\alpha$ -open set U in (X, τ, τ_f) .

Example 4.4 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

$$\text{Let } X = \{a, b, c\} \text{ and } \tau = \{ \phi, X, \{a\} \} \text{ and } \tau_f = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\} \}$$

$$\text{Let } Y = \{1, 2, 3\} \text{ and } \sigma = \{ \phi, Y, \{1\} \} \text{ and } \sigma_f = \{ \phi, Y, \{1\}, \{1, 2\}, \{1, 3\} \},$$

$$\tilde{G}_\alpha O(X) = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\} \}, \text{ and } \tilde{G}_\alpha O(Y) = \{ \phi, Y, \{1\}, \{1, 2\}, \{1, 3\} \},$$

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a strongly $f\tilde{g}_\alpha$ -open.

Proposition 4.5 If a map $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is surjective, $f\tilde{g}_\alpha$ -continuous and $f\tilde{g}_\alpha$ -open, then f is a $f\tilde{g}_\alpha$ -quotient map.

Proof We only need to prove that $f^{-1}(V)$ is open in (X, τ, τ_f) implies V is a $f\tilde{g}_\alpha$ -open set in (Y, σ, σ_f) . Let $f^{-1}(V)$ be an open in (X, τ, τ_f) . Then $f(f^{-1}(V))$ is a $f\tilde{g}_\alpha$ -open set, since f is $f\tilde{g}_\alpha$ -open. Hence V is a $f\tilde{g}_\alpha$ -open set, as f is surjective, $f(f^{-1}(V)) = V$. Thus, f is a $f\tilde{g}_\alpha$ -quotient map.

Example 4.6 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

$$\text{Let } X = \{a, b, c\} \text{ and } \tau = \{ \phi, X, \{a\} \} \text{ and } \tau_f = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\} \}$$

$$\text{Let } Y = \{1, 2, 3\} \text{ and } \sigma = \{ \phi, Y, \{1\} \} \text{ and } \sigma_f = \{ \phi, Y, \{1\}, \{1, 2\}, \{1, 3\} \},$$

$$\tilde{G}_{f\alpha} O(X) = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\} \}, \text{ and } \tilde{G}_{f\alpha} O(Y) = \{ \phi, Y, \{1\}, \{1, 2\}, \{1, 3\} \},$$

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a $f\tilde{g}_\alpha$ -quotient map.

Proposition 4.7 If $f : (X, \tau^{f\tilde{g}_\alpha}, \tau_f) \rightarrow (Y, \sigma^{f\tilde{g}_\alpha}, \sigma_f)$ be a quotient map, then $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is a $f\tilde{g}_\alpha$ -quotient map.

Proof Let V be any open set in (Y, σ, σ_f) , then V is a $f\tilde{g}_\alpha$ -open set in (Y, σ, σ_f) and $V \in \sigma^{f\tilde{g}_\alpha}$. Then $f^{-1}(V)$ is an open in (X, τ, τ_f) . Because f is a quotient map, that is, $f^{-1}(V)$ is a $f\tilde{g}_\alpha$ -open set in (X, τ, τ_f) . Suppose $f^{-1}(V)$ is an open in (X, τ, τ_f) , that is $f^{-1}(V) \in \tau^{f\tilde{g}_\alpha}$. Since f is a quotient map, $V \in \sigma^{f\tilde{g}_\alpha}$ and V is a $f\tilde{g}_\alpha$ -open set in (Y, σ, σ_f) . This shows that $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ is a $f\tilde{g}_\alpha$ -quotient map.

5. Strong form of $f\tilde{g}_\alpha$ -quotient maps

Definition 5.1 Let $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ be a surjective map. Then f is called strongly $f\tilde{g}_\alpha$ -quotient map provided a set U of (Y, σ, σ_f) is an open in Y if and only if $f^{-1}(U)$ is a $f\tilde{g}_\alpha$ -open set in (X, τ, τ_f) .

Example 5.2 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

$$\text{Let } X = \{a, b, c\} \text{ and } \tau = \{ \phi, X, \{a\} \} \text{ and } \tau_f = \{ \phi, X, \{a\}, \{a, b\}, \{a, c\} \}$$

Let $Y=\{1,2,3\}$ and $\sigma=\{\phi, Y, \{1\}\}$ and $\sigma_f=\{\phi, Y, \{1\}, \{1,2\}, \{1,3\}\}$,

$\tilde{G}_{f\alpha}O(X)=\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$, and $\tilde{G}_{f\alpha}O(Y)=\{\phi, Y, \{1\}, \{1,2\}, \{1,3\}\}$,

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a strongly $f\tilde{g}_\alpha$ -quotient map.

Theorem 5.3 Every strongly $f\tilde{g}_\alpha$ -quotient map is $f\tilde{g}_\alpha$ -open.

Proof Let $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ be a strongly $f\tilde{g}_\alpha$ -quotient map. Let V be an open set in (X, τ, τ_f) . Since an every open set is $f\tilde{g}_\alpha$ -open [2] and hence V is $f\tilde{g}_\alpha$ -open in (X, τ, τ_f) . That is $f^{-1}(V)$ is $f\tilde{g}_\alpha$ -open in (X, τ, τ_f) . Since f is strongly $f\tilde{g}_\alpha$ -quotient, $f(V)$ is an open and hence $f\tilde{g}_\alpha$ -open in (Y, σ, σ_f) . This shows that f is a $f\tilde{g}_\alpha$ -open.

Example 5.4 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X=\{a, b, c\}$ and $\tau=\{\phi, X, \{a\}\}$ and $\tau_f=\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y=\{1,2,3\}$ and $\sigma=\{\phi, Y, \{1\}\}$ and $\sigma_f=\{\phi, Y, \{1\}, \{1,2\}, \{1,3\}\}$,

$\tilde{G}_{f\alpha}O(X)=\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$, and $\tilde{G}_{f\alpha}O(Y)=\{\phi, Y, \{1\}, \{1,2\}, \{1,3\}\}$,

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a strongly $f\tilde{g}_\alpha$ -quotient map. Thus f is a $f\tilde{g}_\alpha$ -open.

Remark 5.3 The converse need not be true.

Example 5.4 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X=\{a, b, c\}$ and $\tau=\{\phi, X, \{a\}\}$ and $\tau_f=\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y=\{1,2,3\}$ and $\sigma=\{\phi, Y, \{1\}\}$ and $\sigma_f=\{\phi, Y, \{1\}, \{1,2\}, \{1,3\}\}$,

$\tilde{G}_{f\alpha}O(X)=\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$, and $\tilde{G}_{f\alpha}O(Y)=\{\phi, Y, \{1\}, \{1,2\}, \{1,3\}\}$,

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a $f\tilde{g}_\alpha$ -open. But not 'strongly $f\tilde{g}_\alpha$ -quotient map. Since $f^{-1}(3) = \{c\}$ is not $f\tilde{g}_\alpha$ -open in (X, τ, τ_f) .

Theorem 5.5 Every strongly $f\tilde{g}_\alpha$ -quotient map is strongly $f\tilde{g}_\alpha$ -open.

Proof Let $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ be a strongly $f\tilde{g}_\alpha$ -quotient map. Let V be a $f\tilde{g}_\alpha$ -open set in (X, τ, τ_f) . That is $f^{-1}(f(V))$ is $f\tilde{g}_\alpha$ -open in (X, τ, τ_f) . Since f is strongly $f\tilde{g}_\alpha$ -quotient, $f(V)$ is open and hence $f\tilde{g}_\alpha$ -open in (Y, σ, σ_f) . This shows that f is strongly $f\tilde{g}_\alpha$ -open.

Example 5.6 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X=\{a, b, c\}$ and $\tau=\{\phi, X, \{a\}\}$ and $\tau_f=\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y=\{1,2,3\}$ and $\sigma=\{\phi, Y, \{1\}\}$ and $\sigma_f=\{\phi, Y, \{1\}, \{1,2\}, \{1,3\}\}$,

$\tilde{G}_{f\alpha}O(X)=\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$, and $\tilde{G}_{f\alpha}O(Y)=\{\phi, Y, \{1\}, \{1,2\}, \{1,3\}\}$,

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a strongly $f\tilde{g}_\alpha$ -open.

Remark 5.5 The converse need not be true.

Example 5.6 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y = \{1, 2, 3\}$ and $\sigma = \{\phi, Y, \{2\}\}$ and $\sigma_f = \{\phi, Y, \{2\}, \{1, 2\}, \{2, 3\}\}$,

$\tilde{G}_{f\alpha}O(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$, and $\tilde{G}_{f\alpha}O(Y) = \{\phi, Y, \{2\}, \{1, 2\}, \{2, 3\}\}$,

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a strongly $f\tilde{g}_\alpha$ -open. But not $f\tilde{g}_\alpha$ -quotient map. Since $f^{-1}(\{1\}) = \{a\}$ is not $f\tilde{g}_\alpha$ -open in (X, τ, τ_f) but $\{1\}$ is not open in (Y, σ, σ_f) .

Definition 5.7 Let $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ be a surjective map. Then f is called a $f\tilde{g}^*$ -quotient map if f is $f\tilde{g}_\alpha$ -irresolute and $f^{-1}(U)$ is a $f\tilde{g}_\alpha$ -open set in (X, τ, τ_f) implies U is open in (Y, σ, σ_f) .

Example 5.8 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y = \{1, 2, 3\}$ and $\sigma = \{\phi, Y, \{1\}\}$ and $\sigma_f = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

$\tilde{G}_{f\alpha}O(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$, and $\tilde{G}_{f\alpha}O(Y) = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a strongly $f\tilde{g}^*$ -quotient map.

Theorem 5.9 Every $f\tilde{g}_\alpha^*$ -quotient map is strongly $f\tilde{g}_\alpha$ -open.

Proof Let $f : (X, \tau, \tau_f) \rightarrow (Y, \sigma, \sigma_f)$ be a $f\tilde{g}_\alpha^*$ -quotient map. Let V be a $f\tilde{g}_\alpha$ -open set in (X, τ, τ_f) . Then $f^{-1}f(V)$ is $f\tilde{g}_\alpha$ -open in (X, τ, τ_f) . Since f is $f\tilde{g}_\alpha^*$ -quotient, this implies that $f(V)$ is open in (Y, σ, σ_f) and thus $f\tilde{g}_\alpha$ -open in (Y, σ, σ_f) . Hence f is strongly $f\tilde{g}_\alpha$ -open.

Example- Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y = \{1, 2, 3\}$ and $\sigma = \{\phi, Y, \{1\}\}$ and $\sigma_f = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

$\tilde{G}_{f\alpha}O(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$, and $\tilde{G}_{f\alpha}O(Y) = \{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a strongly $f\tilde{g}_\alpha$ -open.

Remark 5.10 The converse need not be true.

Example 5.11 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y = \{1, 2, 3\}$ and $\sigma = \{\phi, Y, \{2\}\}$ and $\sigma_f = \{\phi, Y, \{2\}, \{1, 2\}, \{2, 3\}\}$,

$\tilde{G}_{f\alpha}O(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$, and $\tilde{G}_{f\alpha}O(Y) = \{\phi, Y, \{2\}, \{1, 2\}, \{2, 3\}\}$,

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a strongly $f\tilde{g}_\alpha$ -open but not $f\tilde{g}_\alpha^*$ -quotient map. Since $f^{-1}(\{1\})=\{a\}$ is $f\tilde{g}_\alpha$ -open in (X, τ, τ_f) but $\{1\}$ is not open in (Y, σ, σ_f) .

Proposition 5.12 Every $f\alpha^*$ -quotient map is a $f\tilde{g}_\alpha^*$ -quotient map.

Proof Let f be a $f\alpha^*$ -quotient map then f is surjective, $f\alpha$ – *irresolute* and $f^{-1}(U)$ is an $f\alpha$ -set in (X, τ, τ_f) implies U is an open set in (Y, σ, σ_f) . Since every $f\alpha$ – *irresolute* map is $f\tilde{g}_\alpha$ -irresolute, $f^{-1}(U)$ is an $f\alpha$ -set which is a $f\tilde{g}_\alpha$. Hence f is a $f\tilde{g}_\alpha^*$ -quotient map.

Example 5.13 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X=\{a, b, c\}$ and $\tau=\{\phi, X, \{a\}\}$ and $\tau_f=\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y=\{1, 2, 3\}$ and $\sigma=\{\phi, Y, \{1\}\}$ and $\sigma_f=\{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

$\tilde{G}_{f\alpha}O(X)=\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$, and $\tilde{G}_{f\alpha}O(Y)=\{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a strongly $f\tilde{g}_\alpha^*$ -quotient map.

Remark 5.14 The converse need not be true.

Example 5.15 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X=\{a, b, c\}$ and $\tau=\{\phi, X, \{a\}\}$ and $\tau_f=\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y=\{1, 2, 3\}$ and $\sigma=\{\phi, Y, \{2\}\}$ and $\sigma_f=\{\phi, Y, \{2\}, \{1, 2\}, \{2, 3\}\}$,

$\tilde{G}_{f\alpha}O(X)=\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$, and $\tilde{G}_{f\alpha}O(Y)=\{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

The function f is defined as $f(a)=1=f(b), f(c)=3$. The function f is a strongly $f\tilde{g}_\alpha^*$ -open but not $f\alpha^*$ -quotient map. Since $f^{-1}(\{1\})=\{a, b\}$ is $f\tilde{g}_\alpha$ -open in (X, τ, τ_f) but $\{1\}$ is not open in (Y, σ, σ_f) .

Proposition 5.16 Every $f\tilde{g}_\alpha^*$ -quotient map is strongly $f\tilde{g}_\alpha$ -quotient map.

Proof If V be an open set in (Y, σ, σ_f) then it is $f\tilde{g}_\alpha$ -open in (Y, σ, σ_f) . Since f is $f\tilde{g}_\alpha$ -irresolute $f^{-1}(V)$ is a $f\tilde{g}_\alpha$ -open in (X, τ, τ_f) . Also V is open in (Y, σ, σ_f) implies $f^{-1}(V)$ is a $f\tilde{g}_\alpha$ -open set in (X, σ, σ_f) . If $f^{-1}(V)$ is a $f\tilde{g}_\alpha$ -open in (X, σ, σ_f) . Since f is a $f\tilde{g}_\alpha^*$ -quotient map, V is an open set in (Y, σ, σ_f) . Hence f is a strongly $f\tilde{g}_\alpha$ -quotient map.

Example 5.17 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X=\{a, b, c\}$ and $\tau=\{\phi, X, \{a\}\}$ and $\tau_f=\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y=\{1, 2, 3\}$ and $\sigma=\{\phi, Y, \{1\}, \{1, 2\}\}$ and $\sigma_f=\{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

$\tilde{G}_{f\alpha}O(X)=\{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$, and $\tilde{G}_{f\alpha}O(Y)=\{\phi, Y, \{1\}, \{1, 2\}, \{1, 3\}\}$,

The function f is defined as $f(a)=1, f(b)=2, f(c)=3$. The function f is a strongly $f\tilde{g}_\alpha$ -quotient map.

Remark 5.18 The converse need not be true.

Example 5.19 Let X be a set and τ be a topology on X and τ_f be a fine topology on X . And Y be a another set and σ be a topology on Y and σ_f be a fine topology on Y .

Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}\}$ and $\tau_f = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $Y = \{1, 2, 3\}$ and $\sigma = \{\phi, Y, \{2\}\}$ and $\sigma_f = \{\phi, Y, \{2\}, \{1, 2\}, \{2, 3\}\}$,

$\tilde{G}_{f\alpha}O(X) = \{\phi, X, \{a\}, \{a, b\}, \{a, c\}\}$, and $\tilde{G}_{f\alpha}O(Y) = \{\phi, Y, \{2\}, \{1, 2\}, \{2, 3\}\}$,

The function f is defined as $f(a)=1=f(b)$, $f(c)=3$. The function f is a strongly $f\tilde{g}_\alpha$ -quotient map but not $f\tilde{g}_\alpha^*$ -quotient map. Since $f^{-1}(\{1\}) = \{a, b\}$ is $f\tilde{g}_\alpha$ -open in (X, τ, τ_f) but $\{1\}$ is not open in (Y, σ, σ_f) .

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